DOCUMENT RESUME

ED 394 812 SE 058 207

TITLE	Pacific Standards for Excellence in Mathematics.
INSTITUTION	Pacific Region Educational Lab., Honolulu, HI.
SPONS AGENCY	Office of Educational Research and Improvement (ED), Washington, DC.
PUB DATE	Sep 95
CONTRACT	R168R20004
NOTE	124p.; For Pacific Standards for Excellence in Science document, see ED 393 692; for Pacific Standards for Excellence in Teaching, Assessment, and Professional Development document, see SP 036 598. Developed by the Pacific Mathematics Leadership Team.
AVAILABLE FROM	Pacific Region Educational Laboratory, Suite 500, 828 Fort Street Mall, Honolulu, HI 96813-4321.
PUB TYPE	Reports - Descriptive (141)
EDRS PRICE	MF01/PC05 Plus Postage.
DESCRIPTORS	Elementary Secondary Education; Learning Activities; *Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; *Standards
IDENTIFIERS	NCTM Curriculum and Evaluation Standards; *Pacific Region

ABSTRACT

This document represents collaborative efforts to define what is valued and necessary in mathematics education for Pacific region children. Underlying assumptions to these standards include constructivism, communication, problem solving, connections, time for learning, and challenge for all students. The majority of the document contains the standards which are grouped into three grade-level sections, K-4, 5-8, and 9-12, and are based largely on the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards. Each standard is described and a sample activity is given. The epilogue discusses curriculum development, textbooks, instruction, alternative assessment, teacher in-service programs, preservice education, and technology. (MKR)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

SE

PACIFIC STANDARDS FOR EXCELLENCE IN MATHEMATICS

DEVELOPED BY THE

PACIFIC MATHEMATICS LEADERSHIP TEAM

ED 394 812

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

PACIFIC MATHEMATICS AND SCIENCE REGIONAL CONSORTIUM
PACIFIC REGION EDUCATIONAL LABORATORY

2

BEST COPY AVAILABLE

SE028207

PACIFIC REGION EDUCATIONAL LABORATORY
828 Fort Street Mall ◆ Suite 500
Honolulu, Hawaii 96813-4321
(808) 533-6000 ◆ FAX: (808) 533-7599
e-mail: askprel@prel.hawaii.edu

Institutional Services
828 Fort Street Mall ◆ Suite 500
Honolulu, Hawaii 96813-4321
(808) 533-6000 ◆ FAX: (808) 533-7599
e-mail: askprel@prel.hawaii.edu

Eastern Pacific Service Center
828 Fort Street Mall ◆ Suite 300
Honolulu, Hawaii 96813-4321
(808) 533-6000 ◆ FAX: (808) 533-7599
e-mail: askprel@prel.hawaii.edu

Western Pacific Service Center
Suite 203, Bank of Hawaii Building
Marina Heights Business Park
PPP 145 ◆ Box 10000
Puerto Rico, Saipan, MP 96950
Phone: (670) 323-6000/1/2
FAX: (670) 323-7735
e-mail: prelwest@prel.hawaii.edu

This publication was produced with funds from the Office of Educational Research and Improvement (OERI), U.S. Department of Education, under grant award number R168R20004. The content does not necessarily reflect the views of OERI, the Department, or any other agency of the U.S. Government.

© Pacific Region Educational Laboratory
September 1995

PACST000001

VISION

ALL PACIFIC CHILDREN WILL BE SCIENTIFICALLY AND MATHEMATICALLY LITERATE:

KNOWLEDGEABLE,

CAPABLE,

AND

CARING.

CONTENTS

Preface v

Acknowledgements vii

About the Pacific Standards for Excellence in Mathematics

Introduction 3
Goals of the Standards 5
Underlying Assumptions 9

Grades K - 4 Standards

1. Mathematics as Problem Solving 15
2. Mathematics as Communication 17
3. Mathematics as Reasoning 19
4. Mathematical Connections 21
5. Estimation 24
6. Number Sense and Numeration 26
7. Concepts of Whole Number Operations 29
8. Whole Number Computation 32
9. Geometry and Spatial Sense 37
10. Measurement 40
11. Statistics and Probability 43
12. Fractions and Decimals 46
13. Patterns and Relationships 49

Grades 5 - 8 Standards

1. Mathematics as Problem Solving 53
2. Mathematics as Communication 55
3. Mathematics as Reasoning 57
4. Mathematical Connections 59
5. Number and Number Relationships 61
6. Number Systems and Number Theory 64
7. Computation and Estimation 67
8. Patterns and Functions 70
9. Algebra 72
10. Probability and Statistics 74
11. Geometry 76
12. Measurement 79

Grades 9 - 12 Standards

1. Mathematics as Problem Solving 83
2. Mathematics as Communication 85
3. Mathematics as Reasoning 87
4. Mathematical Connections 89
5. Algebra 91
6. Functions 93
7. Geometry 95
8. Geometry from an Algebraic Perspective 97
9. Trigonometry 99
10. Statistics 101
11. Probability 103
12. Discrete Mathematics 105
13. Conceptual Underpinnings of Calculus 108
14. Mathematical Structures 110

Epilogue

Next Steps: Our Challenge 115
Obtaining More Information About the Standards 118
References 120

PREFACE

In 1992, the Pacific Mathematics and Science Regional Consortium was formed with funding from the U.S. Department of Education's Dwight D. Eisenhower National Program for Mathematics and Science Education. Headquartered in Honolulu, Hawaii, at the Pacific Region Educational Laboratory (PREL), the consortium is a collaboration of PREL, the University of Hawaii's Curriculum Research & Development Group, the Moanalua Gardens Foundation, and the Departments of Education of the region's ten entities: American Samoa; Commonwealth of the Northern Mariana Islands; Federated States of Micronesia (Chuuk, Kosrae, Pohnpei, and Yap); Guam; Hawaii; Republic of the Marshall Islands; and Republic of Palau.

One of the first tasks the Consortium undertook was the development of standards in mathematics and science appropriate to the Pacific region. The resulting documents, products of the Pacific Mathematics and Science Leadership Team, contain challenging standards for the Pacific region.

The Pacific Standards for Excellence Series

The Pacific Standards for Excellence Series currently consists of this document and two others: The *Pacific Standards for Excellence in Science* and the *Pacific Standards for Excellence in Teaching, Assessment and Professional Development*. These documents are the result of wide-spread collaboration and will set ambitious goals for mathematics and science education in the region. They are targets for students, teachers, and schools to strive for.

The *Pacific Standards for Excellence in Mathematics* and *Pacific Standards for Excellence in Science* identify what all students should know, be able to do, and care about as a result of their education. The *Pacific Standards for Excellence in Teaching, Assessment and Professional Development* describe the teaching, professional development, and assessment necessary to create learning environments which are supportive of students who are striving to achieve mathematical and scientific literacy.

All three documents are firmly based upon research and similar work of outstanding groups engaged in mathematics and science education reform. The standards contained in the Pacific Standards for Excellence Series are challenging while acknowledging the Pacific region as a variety of environments, cultures, and experiences. These are "living" documents which will continue to be refined and developed as they are implemented.

The Pacific Standards for Excellence in Mathematics

The *Pacific Standards for Excellence in Mathematics* represents the Pacific mathematics educators' response to the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989). As the Pacific Mathematics Leadership Team, we identified goals for the Pacific region. We also reviewed the NCTM document with the Pacific region's environments, cultures, and needs in mind. Relevant recommendations were adapted from the NCTM document or new standards were created especially for the Pacific region.

This document is primarily for teachers, administrators, curriculum planners, and other educators. As school staffs, district personnel, and other groups propose solutions to curricular problems and questions, these standards should be used as criteria against which to judge their ideas. They should also be used to plan for staff development, facilities improvement, and technology implementation. Additionally, the standards are useful for community leaders, legislators, parents, and others concerned with education. Finally, we hope that the document may be useful to others outside our region who are interested in and concerned about mathematics education and the development of mathematically literate citizens.

ACKNOWLEDGMENTS

These *Standards* represent the Pacific mathematics educators' response to the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989) and were adapted from that document. This document is the result of work by each of us who represent every entity in the region, the Curriculum Research and Development Group (CRDG) at the University of Hawaii, and the Pacific Eisenhower Mathematics and Science Regional Consortium at the Pacific Region Educational Laboratory(PREL).

We recognize the critical role of the Pacific Mathematics and Science Regional Consortium Advisory Board in this effort and thank its members for providing guidance and support to the Leadership Team members and consortium staff. We also acknowledge the efforts of the following individuals who made significant contributions to the development of the *Pacific Standards for Excellence in Mathematics*:

Tangy Anastacio, Ministry of Education, Republic of Palau
Alice Borja, PREL, Program Specialist
Winona Chang, PREL, Program Specialist
Pernis Diopulos, Department of Education, Pohnpei, Federated States of Micronesia
Barbara Dougherty, Associate Professor, CRDG, University of Hawaii
Ginny Fenenigog, Department of Education, Yap, Federated States of Micronesia
Philo Jennings, Department of Education, American Samoa
Bedinin Joseph, Ministry of Education, Republic of the Marshall Islands
Selihter Joseph, Department of Education, Pohnpei, Federated States of Micronesia
John Kitchar, Public School System, Guam
Jenny McCrory, Junior Specialist, CRDG, University of Hawaii
Teresita Mesa, Public School System, Guam
Gibson Mongkeya, Department of Education, Kosrae, Federated States of Micronesia
Jeanne Nelson, Kamehameha Schools, Hawaii
Kathleen Nishimura, Department of Education, Hawaii
April Olkeriil, Ministry of Education, Republic of Palau
Nicolas Sablan, Public School System, Commonwealth of the Northern Mariana Islands
Sanfio Sony, Department of Education, Chuuk, Federated States of Micronesia

We gratefully acknowledge the work of the National Council of Teachers of Mathematics and the *Curriculum and Evaluation Standards for School Mathematics* (1989) upon which the Pacific Standards for Excellence in Mathematics are based.

About the Pacific Standards for Excellence in Mathematics

INTRODUCTION

Mathematics is the key to opportunity. It is no longer just the language of science, finance, economics, health, or medicine only. Mathematics can maximize our students' opportunities and open doors to careers. As they grow into productive citizens, our young people are empowered to make informed decisions about their own lives, their government, and their global society.

The *Pacific Standards for Excellence in Mathematics* provides a vision of what is appropriate school mathematics today and in years to come. Its purpose is to lay the foundation for the Pacific child's mathematics education program that will build tools to help the child become a contributing member of society. To better prepare today's students for tomorrow, our schools, teachers, students, parents, and the community-at-large must raise the level of mathematics education and associated expectations.

These standards were created with the whole child in mind. In adapting the NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989), we, the Leadership Team, made a concerted effort to illustrate the *Standards* with student-centered activities. You will notice that with each standard in each of the grade levels, there is a short description of the standard and its intent and then an activity is suggested for implementing the standard at that level. References and handouts are included.

The *Pacific Standards* advocates a shift away from routine work that generally consists of algorithms and step-by-step methods to mathematics grounded in problem solving and higher-order thinking. This shift in mathematics content will require new teaching methods that are more student-centered. The combination of new mathematics and instructional methods will provide a more meaningful mathematics program for students and motivate them to continue studies in upper-level mathematics courses.

The *Pacific Standards for Excellence in Mathematics* represents our collaborative efforts to define what is valued and necessary in mathematics. Teachers and schools may use it as a guide for examples of new or unfamiliar mathematical topics. Governments can use it to align their curriculum frameworks or guides with a contemporary view of mathematics education. Mathematics educators can adapt or develop assessment programs to reflect the changes in mathematics. Finally, this document can be the basis for staff development components.

STANDARDS

Standards are statements about what is valued in mathematics education. They define the global ideas important in a well-rounded mathematics education program. In the Pacific *Standards*, there is a broad representation of mathematical topics, rather than a narrow focus on computation or number ideas.

Standards provide criteria or characteristics against which the quality of materials, other curriculum documents, and classroom practices can be compared. Educators can compare mathematics materials with the *Standards* to evaluate how closely the materials are aligned with the expectations.

A **standard** is a statement about what is valued. It forms criteria by which to judge excellence.

It is hoped that you will take these standards and bring them to life in your classroom. It is through this life that our Pacific children can reach the vision.

GOALS OF THE STANDARDS

The goals for the *Pacific Standards* were established with the whole child in mind. They represent what is necessary for our children to be successful in a technological world that changes very quickly, often in ways we could not envision. We must, therefore, educate our students, so that they are capable of meeting new challenges.

The goals of the *Pacific Standards* reflect qualities needed for success in an ever-changing world.

The study of mathematics includes intellectual and aesthetic pursuits. Intellectually, students investigate and explore problem-solving tasks in numerical, statistical, geometrical, and algebraic contexts. Rather than the memorization of step-by-step methods, these tasks encourage students to make their own conjectures and test their hypotheses. New mathematical understandings build on their findings from these explorations. Most importantly, however, students communicate their understandings in their own language instead of language from the textbook. The content knowledge is thus richer and more meaningful to students.

Aesthetically, students discover relationships between the environment and geometrical and numerical concepts. The representation of mathematical concepts in nature and the environment connects learning for children. Additionally, students gain a respect for the beauty of mathematical reasoning, as they engage in problem-solving tasks that are motivating.

The problem-solving focus represented in the *Standards* and associated activities will help students to function in an ever-changing workplace. Even though the problem-solving tasks are mathematical, the solution strategies are used in everyday situations that require decisions to be made. Predicting, sorting out needed from unimportant information, and analyzing solutions in the context of a situation are important characteristics of decision making.

As the Pacific Mathematics Leadership Team drew up the *Standards*, our vision for our students guided our thinking. We first looked at the needs of a demanding work force and society to determine the characteristics of a successful citizen. From there, we identified three goals.

Goal 1: Mathematically literate workers can:

- find multiple solutions to problems.
- independently problem solve.
- make new applications with their knowledge.
- retrain for a new job(s).

Goal 2: Lifelong learners can:

- adapt to a changing workplace.
- work independently or with a team.
- make new applications with their knowledge.
- have a well-rounded, fulfilling life.
- use thinking tools that can help solve problems not now in existence.
- develop an attitude for learning beyond the classroom walls, beyond the school day.

Goal 3: Informed decisions are made by students who:

- are aware of governmental, religious, economic, and other social trends and beliefs.
- understand complex information.
- understand environmental changes and their associated implications.
- can apply mathematics to jobs, everyday events, and personal lives.

Students can productively contribute to society as mathematically literate workers, lifelong learners, and informed decision makers.

The Leadership Team was also aware that, for students to accomplish the three goals, **all** students must have access to important mathematics. Students should not be separated into groups that receive different content or for whom there are different expectations. We, therefore, added a fourth goal to these *Standards*.

Goal 4: Open mathematics to all students, so that they have opportunities to:

- learn and be intellectually challenged.
- achieve in mathematics with a belief that success does not depend on innate talent.
- develop mathematical power.

For students to be successful citizens, they must first be given the opportunity to learn.

These goals are characterized by dramatic shifts in mathematics education related to content, instruction, and assessment. The content that was once reserved for the top few is now accessible to all students and includes such topics as probability, statistics, discrete mathematics, transformational geometry, functions, and algebra. The instructional methods use student language as the means to develop ideas about the topics. Finally, students demonstrate their mathematical understandings in a variety of ways including projects, journal writings, problem creations, and discussion.

These shifts will require teachers to rethink their concepts about how students learn and what instructional methods promote learning. Lecture and practice have long been the primary instructional approach in mathematics from elementary through secondary classes. The result has been students who are dependent upon the teacher to supply the "correct" method to solve problems and who have little creativity in nonroutine problem solving. With new demands on our students as they enter the work force, we must restructure our schools to provide experiences in mathematical content that extend beyond what has traditionally been a part of mathematics classes.

Shifts in Mathematics Education

From a Traditional Model	To a Contemporary One
Mathematics for some	Mathematics for all
Homogeneous classes	Heterogeneous classes
Individual work	Collaborative efforts
Skill development, manipulation of symbols, and vocabulary before problem solving	Problem-solving approach to instruction that uses student language
Students learn from teacher	Students learn from each other, teacher, and other resources
Reliance on memorized facts	Focus on understanding
Passive learning	Active learning
Fragmented topics	Connected topics
Emphasis on arithmetic	New definition of basic skills
Short exposure to topics	Spiraled throughout
Paper/pencil test	Multidimensional assessment

UNDERLYING ASSUMPTIONS

The traditional mathematics class has been taught with the predictable lecture and drill approach. Students are given specific methods to solve problems and then practice each technique on a large number of problems. The emphasis is placed on getting the *one* correct answer and following the teacher's steps in the solution process. Even though this is a common teaching method, there is much evidence to support its ineffectiveness in promoting long-term learning and solid understanding of concepts.

Constructivism

Constructivism is the belief that students build their own knowledge by integrating new information and ideas into existing knowledge structures. When there are inconsistencies between the new and old ideas, the learner analyzes what he/she believes to be true against new ideas. In the process, some old ideas may be replaced or restructured with new notions or new ideas discarded as invalid.

Learning occurs when students build their own understandings by making connections among new ideas and existing knowledge.

Constructivism has several implications for teaching. First, learning does not occur unless students perceive discrepancies. They must have the opportunity to explore tasks that provide tension or dissonance with their "old" knowledge.

Second, the idea that all students leave the classroom with the same information if the teacher gives clear and succinct explanations is not valid. No matter the detail, students put new ideas into their already existing knowledge. They may not perceive what teachers say in the same way that teachers meant it to be.

Third, students need to share ideas in order to validate or refute their understandings or beliefs. They should have opportunities to discuss their own ideas and analyze others. The instructional approach changes from one of "teacher as authority" to "teacher as facilitator." Additionally, mathematical tasks should be open-ended with multiple solutions and solution strategies rather than close-ended problems with only one correct answer and a step-by-step solution method. These open-ended tasks motivate discussion and encourage exploration.

Finally, teachers must acknowledge that students structure and integrate their knowledge through a variety of modalities, such as linguistic, spatial, mathematical and musical. That is, students have different strengths and need to have a variety of teaching strategies used in the class in order to find a modality or method that fits how they structure their knowledge.

Constructivism supports five specific premises (Vygotsky, 1978) that optimize learning experiences. These include communication, problem solving, connections within and outside mathematics, time for learning, and challenges for all students.

Communication (social interaction), problem solving, connections within and outside of mathematics, time for learning, and open-ended challenges optimize learning experiences for all students.

Communication. Social interaction is the means by which we convey our ideas and make new conjectures. For many of us, social interaction conjures up visions of students discussing after-school activities, but it has a different connotation in a classroom that emphasizes student learning. Social interaction provides the means for students to use their language to communicate original ideas about mathematics.

Social interaction appears in at least five forms. First, the most obvious form of social interaction is oral discussion. Students describe, verify, and challenge solution methods and results of investigations. The teacher acts as the facilitator, asking questions that probe, clarify, or challenge student ideas.

Second, social interaction is accomplished through writing tasks. Students can be given journal prompts and their responses are indicative of their level of mathematical development. Journal prompts can relate to the mathematical content, to student feelings about themselves as mathematicians and students of mathematics, to student views about mathematics as a discipline, and to metacognitive aspects about solution processes students use. Additionally, students can create new problems for other students to solve or they can write detailed descriptions about how they solved a nonroutine problem.

Third, social interaction occurs through listening. As students share their ideas, other students must develop listening skills, so that they can analyze what others are saying. Their analysis forms the basis for refuting or verifying others' ideas.

Reading is the fourth form of social interaction. Mathematics classes can use resources other than the textbook. These may include trade books, reference books, or student-created problems. Students can use the resources as support for their ideas, much as one would in the workplace.

Finally, building models is another form of communication. Models can be constructed from commercial materials or from any items that students feel communicate important characteristics of a mathematical topic.

Problem solving. In the past, students learned skills first and, only after they had mastered algorithms, were they given the opportunity to engage in problem-solving tasks. The learning of skills through routinized methods did not promote student

creativity nor did it help students learn to attack nonroutine or unfamiliar problems. These standards provide a different approach to problem solving. That is, they recommend that students first be exposed to problem solving and, from these experiences, move to skills.

Problem-solving tasks allow students to bridge what they have experienced into new areas. Good problems give students the opportunity to use whatever they feel comfortable with in the solution process. Thus, students may choose a variety of problem-solving strategies that best fit their way of learning.

Additionally, students develop a growing knowledge about how they are learning. Many adults have never been "in touch" with their thinking as they solve problems. They may not realize that they gravitate to certain solution processes. This knowledge helps students become better problem solvers.

Connections. Closely associated with problem solving is connections, a linking between old and new knowledge. As students confront new problem contexts, they look to their old experiences to help define ways to solve the new problems. The fluidity of problem contexts develops the strong connections.

It is important for students to form connections among their knowledge bases. Mathematics has been typically taught as a set of fragmented and isolated skills that students forget when a new skill is introduced. The connections among mathematical topics help students determine the rationale behind particular algorithms and, hence, solidify the learning of otherwise meaningless mathematics.

Time for learning. For decades, mathematics teachers have drilled students in computational methods. The primary method of learning was to practice, practice, and then do more practice. If, however, a teacher were working with a "faster" student, one who scored high on a standardized test, then perhaps the pace was faster.

We now know that regardless of ability level, students need approximately three to eight days to really understand a new idea. That does not mean that it is the only topic that students study during that time but it does imply that students should see problems related to a topic over a period of days and not for just one night. This supports the call for more in-depth work in concepts rather than a superficial treatment of a topic.

Challenge for all students. One of the goals of the *Standards* is to allow **all** students the opportunity to succeed in mathematics. Veteran and inexperienced teachers often claim that it is not possible to challenge all students in the same class. One method that does encourage students to work at their own level is the use of open-ended questions.

Open-ended questions allow students to rise to the height of their understandings in order to respond. Multiple-solution responses encourage students to continue to find more complex answers. And, because there are multiple answers, students persevere on the problems longer.

The result is that students become more confident of their mathematical abilities and they are more willing to share their responses. Challenged students are more engaged in class discussions and look forward to accepting new and bigger challenges.

Grades K - 4
Standards

Standard 1: Mathematics as Problem Solving

In grades K–4, the study of mathematics should emphasize problem solving so that students can—

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from everyday and mathematical situations;
- develop and apply strategies to solve a wide variety of problems;
- verify and interpret results with respect to the original problem; and
- acquire confidence in using mathematics meaningfully.

Problem solving can be thought of in at least two ways. First, problem solving represents a method of instruction where teachers ask process-type questions that encourage students to explore. These questions may include these: Can you solve the problem another way? Is there another solution? How can you justify your solution process? and How does this problem relate to others that you've solved? Process or problem-solving questions require extended answers and motivate students to look at the mathematics in a deeper fashion.

Problem solving also suggests the type of problem that students solve. The standards advocate problems that require an extended time period to solve, that are solved with problem-solving strategies (working backwards, make a table, guess-and-test, and so on), and that can be extended to probe a concept or idea even further. These problems do not lend themselves to solutions by applying computational methods. They promote higher-level thinking and creative solution techniques.

Standard 1: Problem-solving Activity

Standard's bullets addressed by activity

- *use problem-solving approaches to investigate and understand mathematical content*
- *verify and interpret results with respect to the original problem*
- *develop and apply strategies to solve a wide variety of problems*

Materials needed

- 5 X 5 geoboards and geobands

Description of Activity

- **Group or classroom management practices**

  Children work in pairs or small groups.

Have students make a triangle on the geoboard. How many pegs does the rubber band touch? Make a triangle so that the rubber band touches three pegs. Make another triangle so that the rubber band touches four pegs, and another triangle that touches five pegs. Can you make a triangle so that the rubber band touches more pegs? Is it possible to make a triangle that touches only two pegs? Is it possible to make a triangle that has the same number of pegs on two sides? On three sides?

Make a square with the rubber band touching four pegs and a square with the rubber band touching eight pegs. What other squares can you make? Count the number of pegs on each side. Discuss why each side must have the same number of pegs. Establish a pattern with the total number of pegs on the boundary of a square: 4, 8, 12 ...

Explore rectangles the same way. The number of pegs may not be the same for all four sides but will be the same for opposite sides. Once the children have grasped the idea of pegs on the boundary, have them illustrate what is meant by "pegs inside." Although the pegs touching the rubber band are technically "inside," for this problem, count only those pegs that are not touching the rubber band.

Ask the children to make a triangle with two pegs inside, then with three pegs inside. What is the greatest number of pegs that can be inside a triangle on a geoboard?

Repeat these problems using other figures, such as four-sided figures and six-sided figures. Is it possible to get more pegs inside a four-sided or a five-sided figure? Note that the answers will vary according to the figures the children make.

Reference

Burton, G., Clements, D., Coburn T., Del Grande, J., Firkins, J., Joyner, J., Leiva, M., Lindquist, M., & Morrow, M. (1991). *Curriculum and evaluation standards for school mathematics addenda series, Grades K-6: First-grade book.* Reston, VA: National Council of Teachers of Mathematics. pp. 20-21.

Standard 2: Mathematics as Communication

In grades K–4, the study of mathematics should include numerous opportunities for communication so that students can—

- relate physical materials, pictures, and diagrams to mathematical ideas;
- reflect on and clarify their thinking about mathematical ideas and situations;
- relate their everyday language to mathematical language and symbols; and
- realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics.

Communication includes at least five instructional methods: 1) writing, 2) speaking, 3) reading, 4) listening, and 5) constructing or modeling. Writing can involve responses to journal prompts, problems created by students, and descriptions of problem solutions. Speaking is that interaction done by the students as they explain their thinking to other students and the teacher. It is important to note that it is the **students** who are talking and not just the teacher. Reading refers to students reading each others' work or problems or using other resources. Listening allows students an opportunity to critically analyze other students' ideas. Finally, as students construct shapes or model concepts with concrete materials, they communicate ideas about a mathematical topic.

Standard 2: Communication Activity

Standard's bullets addressed by activity

- *reflect on and clarify their thinking about mathematical ideas and situations*
- *relate their everyday language to mathematical language and symbols*
- *realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics*

Materials needed

- Paper money/coins (optional)

Description of Activity

- **Group or classroom management practices**

Students work in groups of four.
For each situation, write how you would feel about acquiring money in that way and how you would use it.

- Situation 1: Your grandparents gave you $50.00 for your birthday.
- Situation 2: You earned $50.00 from your parents for doing your chores.
- Situation 3: You found $50.00 on the school playground.
- Situation 4: You earned $50.00 for weaving two baskets for the handicraft shop.

(Note: Students may brainstorm other situations.)

Each person in a group of four members should choose a different situation to write about. After they have written about their particular situations, they will discuss their "solutions" first in their own small groups and then as a class.

Some questions to ask in the large group may include:

- Did you feel comfortable about acquiring the money in the situation you chose?
- Justify your answer.
- Now that you have heard other people's "solutions" do you think you will change how you chose to spend or invest the money?

- **Extensions**

Lesson can be modified to be more specific or grade-level appropriate by varying the situations and amounts so that they are more realistic for that grade level.

Students may also write about how they would spend the money they have saved for a year. Will they use it for themselves, give it away to charity, or spend it or invest it in other ways?

Reference

Winocur, S. (1986). *How to spend it.* California: IMPACT. p. 4, lesson 2.10.2.

Standard 3: Mathematics as Reasoning

In grades K–4, the study of mathematics should emphasize reasoning so that students can—

- draw logical conclusions about mathematics;
- use models, known facts, properties, and relationships to explain their thinking;
- justify their answers and solution processes;
- use patterns and relationships to analyze mathematical situations; and
- believe that mathematics makes sense.

Critical thinking is the heart of mathematics. In fact, mathematical problems cannot be solved without reasoning the solution process and deciding if the answer is logical. This standard does not refer to the formal reasoning process, but it should portray the conjecturing, validating, refuting, and analyzing processes that are used to make sense of mathematics. Students can use various means to communicate their reasoning, such as oral discussion, construction of models, and written explanations.

Standard 3: Reasoning Activity

Standard's bullets addressed by activity

- *use models, known facts, properties, and relationships to explain their thinking*
- *justify their answers and solution processes*
- *use patterns and relationships to analyze mathematical situations*

Materials needed

- Students will decide in their groups how to best collect the data.

Technology

- Calculators are optional but highly recommended.
- Computers with software that have graphing capabilities are optional but highly recommended.

Description of Activity

- **Group or classroom management practices**

  Kosrae has 4 villages: Lelu, Tafunsak, Malem, and Utwe. Students should work in small groups of three or four members, grouped according to where they live.

Each group needs to specify in which section of their village they will collect data. They will spend five minutes by the road on each of seven consecutive days to record the number and the color of cars and trucks that pass by within this time frame.

Students will decide for themselves how to best collect the data. Some things they need to consider on their own may include:

- What part of the village would be best to collect data?
- When is the best time to collect this information?
- Should we collect this information at the same time everyday? If so, why is this important? If not, why isn't it important?
- How should we record our information? Would making a table to keep track of our tallies be helpful?

After collecting the data each group will decide how best to report the information. Some decisions they need to make on their own may include:

- What is the best way to share the information with the class?
- If we decide to graph the data, what type of graph would be best?
- Could we report the information in other ways?

Different groups share their data with the rest of the class. What kinds of facts or inferences can the class conclude from each group's report? Some of these may include:

- Are there more cars or trucks in their village? How can we verify it?
- If trucks are more popular, why is this so?
- Is there a particular color of vehicle that is most popular? Why is this so?
- How useful are the data we gathered? Who would use the data?

- **Extensions**

This activity can be modified for a specific grade-level. Other kinds of information can be collected, for example, the amount of rainfall each month, the types of crops grown in a village, the number of tourists that visit the island each month, and so on.

Reference

Kosrae Department of Education: Kosrae, Federated States of Micronesia.

Standard 4: Mathematical Connections

In grades K–4, the study of mathematics should include opportunities to make connections so that students can—

- link conceptual and procedural knowledge;
- relate various representations of concepts or procedures to one another;
- recognize relationships among different topics in mathematics;
- use mathematics in other curriculum areas; and
- use mathematics in their daily lives.

Mathematics is often thought of as separate topics that are not intertwined but are studied in isolation. Connecting the topics helps students form a more cohesive knowledge base. This eliminates the memorization of facts or procedures that appear to have little meaning.

Connections also relate directly to how students learn mathematics. Students who make connections usually explore a mathematical topic for a period of days and then bridge ideas that are represented in concrete form to a more abstract one. Additionally, they are able to discern similarities and differences among ideas and procedures.

Standard 4: Connections Activity

Standard's bullets addressed by activity

- *use mathematics in their daily lives*

Materials needed

- Real or play coins (pennies to quarters)
- Coin Count Recording Sheet (included)

Description of Activity

- **Group or classroom management practices**

  Students work in pairs.

Give each group a set of coins—pennies, nickels, dimes, and quarters. In situational contexts, ask students to make different combinations of various sums. For example:

> Maria bought a can of soda at a local store for 65 cents. Find as many combinations of coins that she could have used to buy the soda. (Students record their answers on the recording sheet.)

Suggested questions:

- How many possible combinations are there?
- Can you predict how many combinations are possible?

Other situations could involve giving correct change. For example: Jose bought a popsicle for 55 cents. He gave the cashier a dollar. What possible combinations of coins could the cashier give him for his change?

- **Extensions**

Have students create a restaurant or store and design a menu or price list. Older students can develop word problems to go with their projects. Students can go to the restaurant or store and practice their estimation skills, mental math skills, and computational skills. Calculators may be used to verify their answers.

Reference

Goodman, J. (1992). *Group solutions: Cooperative logic activities for grades K-4*. Berkeley, CA: Lawrence Hall of Science, University of California - Berkeley. pp. 94-97.

Coin Count Recording Sheet

Name(s)_____

Coin Count #	Quarters	Dimes	Nickels	Pennies	Total Money

Goodman, J. (1992). *Group solutions: Cooperative logic activities for grades K-4*. Berkeley, CA: Lawrence Hall of Science, University of California - Berkeley. p. 111.

Standard 5: Estimation

In grades K–4, the curriculum should include estimation so students can—

- explore estimation strategies;
- recognize when an estimate is appropriate;
- determine the reasonableness of results; and
- apply estimation in working with quantities, measurement, computation, and problem solving.

Estimation is a lifelong skill, necessary for the technological society. It helps students develop flexibility as they work with numbers and measurements, leading to better judgments about reasonableness and sensibleness of results. As technology plays a bigger role in their activities, students should be able to judge whether or not their answer is appropriate with regard to a range, the number of digits, and the type (whole or decimal number).

Estimation should not be thought of only as rounding. There are many estimation strategies that students naturally use such as front-end, chunking, and special or compatible numbers. Estimation activities should have a context, not the typical "estimate then compute" computation problems. Without a context, students do not develop an awareness of when an estimate is appropriate rather than an exact answer. The context also determines whether an under- or over-estimate is more appropriate.

Standard 5: Estimation Activity

Standard's bullets addressed by activity

- *explore estimation strategies (in numerosity)*
- *apply estimation in working with quantities, measurement, computation, and problem solving*

Materials needed

- Shells, stones, or any discrete objects

Technology

- Some students may use a calculator once they find a referent.

Description of Activity

- **Group or classroom management practices**

  Students work individually or in pairs.

Fill a jar or any transparent container with shells, stones, or other objects. The objects should be about the same size and shape. Set the container in an area where students can examine it. They are to predict the number of items in the container by writing their predictions on a sheet of paper. Below their prediction, ask them to write an explanation of how they arrived at their estimate. What strategies did they use? How can they verify their predictions?

- **Extensions**

Display a shell, stone, or other discrete item for students to examine for a few days along with some empty containers. Ask students to predict the size of the container needed to hold 100 (or other quantity) of the item. (Be sure you have enough of the item for them to check out their predictions at the conclusion of the activity.) Include the students' suggestions on how to use the item as a "referent" in making predictions when the class discusses the activity.

Standard 6: Number Sense and Numeration

In grades K–4, the mathematics curriculum should include whole number concepts and skills so that students can–

- **construct number meanings through real-world experiences and the use of physical materials;**
- **understand our numeration system by relating counting, grouping, and place-value concepts;**
- **develop number sense; and**
- **interpret the multiple uses of numbers encountered in the real world.**

Number sense encompasses several areas of mathematics that would indicate a good "feel" for numbers and operations. There are five components of number sense: 1) number meanings, 2) number relationships, 3) magnitudes of numbers, 4) effects of operations, and 5) measurement referents.

Number meanings refer to both cardinal and ordinal numbers. It is not rote counting but a feel for the "numberness" of a word that expresses a quantity. When students hear the word "twenty," they should "see" a set of items in their mind.

Number relationships are formed by the composition and decomposition of numbers. For example, 20 can be thought of as 2 tens, 1 ten and 10 ones, or 20 ones. Further, 9 can be represented as one less than 10, two more than seven, or an odd number.

Magnitudes of numbers are expressed in various relationships. Fifty-two can be thought of as close to half of 100 or small compared to 500. Place value concepts help to develop students' ideas about number magnitude.

The effects of operations on numbers is important in helping students develop notions about the reasonableness of answers. Students may make comparisons about what happens when you multiply two whole numbers versus when you multiply two numbers between 0 and 1.

Measurement referents are not solely related to geometric measurements. Students should be able to judge the appropriateness of units of answers as well as the magnitude. It is impossible, for example, for someone in third-grade to be 3 meters tall or to weigh 12 kg.

Standard 6: Number Sense and Numeration Activity

Standard's bullets addressed by activity

- *construct number meanings through real-world experiences and the use of physical materials*
- *understand our numeration system by relating counting, grouping, and place-value concepts*
- *develop number sense*

Materials needed

- Counters such as dry beans, small shells, small stones, color chips, or the like
- Teddy's Mat activity sheet (included) or something similar

Description of Activity

- **Group or classroom management practices**

  Students work in pairs.

Ask the first partner to place two counters on one side of Teddy's Mat while counting aloud. Tell the second partner to put three counters on the other side of the mat while counting aloud. Which side has more? Ask students to show a justification for their answer.

Repeat the activity, using different numbers. Allow different children to tell about their counters, and continue to ask questions:

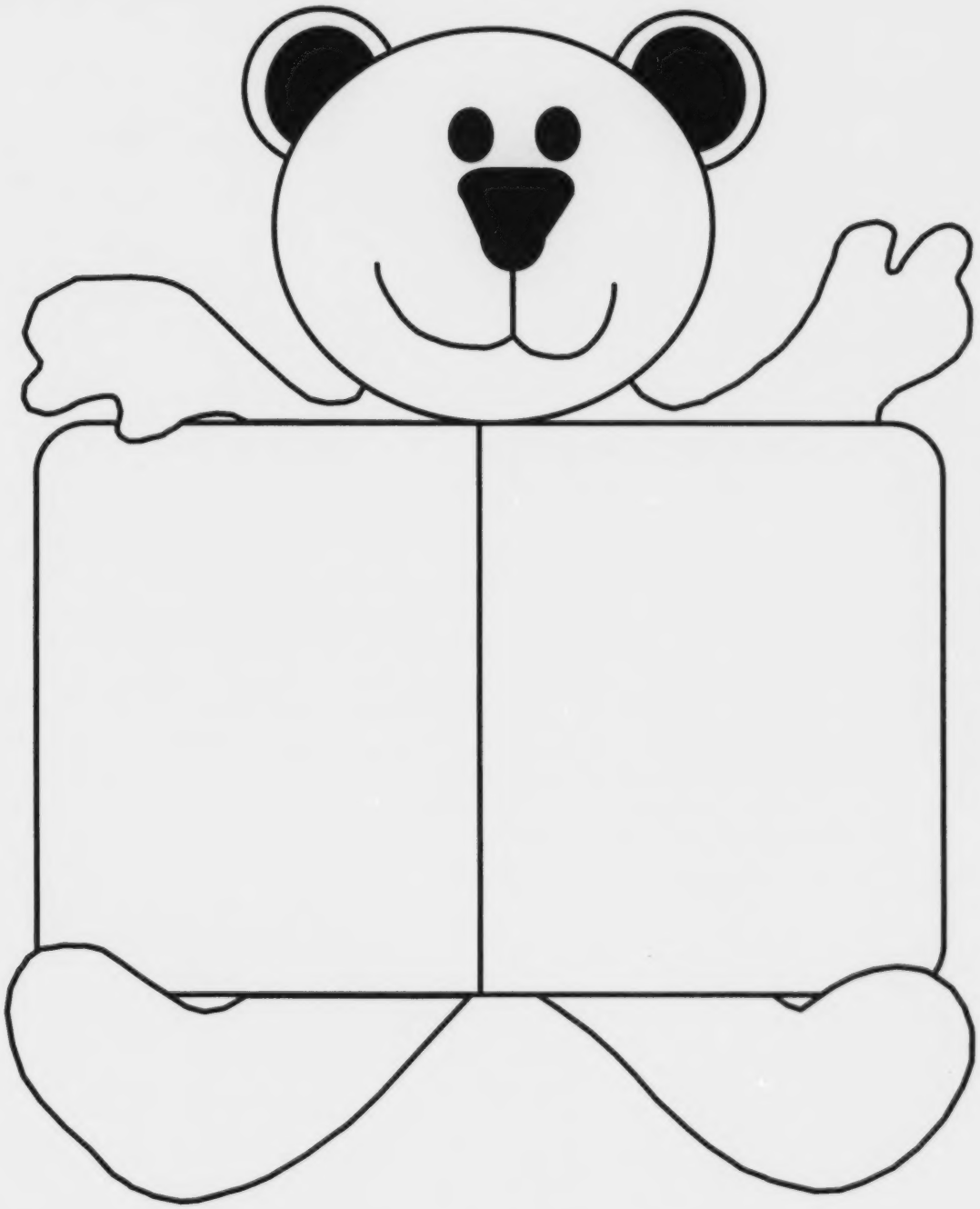
- The first person has five counters. What number could the second person place on the other side so that there would be fewer?
- If you put seven counters on the other side, would that be more or fewer than five?
- If we want to have the same number of counters on each side, who can tell what we might do?

- **Extensions**

Use this activity as a mathematics learning center game by having the children take turns rolling a die and placing that many counters on their side of Teddy's Mat. The children then compare to see who has more counters, fewer counters, or the same number of counters.

Reference

Burton, G., Coburn, T., Del Grande, J., Lindquist, M., Morrow, L., Clements, D., Firkins, J., & Joyner, J. (1991). *Curriculum and evaluation standards for school mathematics addenda series. Grades K-6: Kindergarten book.* Reston, VA: NCTM. pp. 9, 12.



Standard 7: Concepts of Whole Number Operations

In grades K–4, the mathematics curriculum should include concepts of addition, subtraction, multiplication, and division of whole numbers so that students can—

- **develop meaning for the operations by modeling and discussing a rich variety of problem situations;**
- **relate the mathematical language and symbolism of operations to problem situations and informal language;**
- **recognize that a wide variety of problem structures can be represented by a single operation; and**
- **develop operation sense.**

Concepts of whole number operations refer to an understanding of how and why operations work the way they do. Students work with models of operations that demonstrate why algorithms were developed. The models generally require that students use concrete materials or manipulatives in their demonstrations.

The concepts of operations are based on students' intuitions about how to solve problems from real contexts. That is, if two children were picking mangoes, to find how many they had altogether, they would put the two sets (or collections) of mangoes together and count them. This is a model of addition known as "joining."

Standard 7: Whole Number Operations Concept Activity

Standard's bullets addressed by activity

- *develop meaning for the operations by modeling and discussing a rich variety of problem situations*
- *relate the mathematical language and symbolism of operations to problem situations and informal language*
- *recognize that a wide variety of problem structures can be represented by a single operation*

Materials needed

- Base Ten Blocks

Description of Activity

- **Group or classroom management practices**

  Students work in pairs or in small groups of three to four members.

Using Base Ten Blocks or other manipulatives, show how you would solve each problem:

Take Away

- John has 96 baseball cards. He gave 32 cards to Paul. How many cards does he have now?

- There were 978 people at a football game. Because of the heavy rain, 695 people left the game early. How many people braved the rain to continue watching the game?

- Smelly found 24 snails for the picnic. Only 18 were eaten. How many were left?

Comparison

- Children have 20 teeth. Adults have 32 teeth. How many more teeth do adults have?

- Kimo is 57 inches tall. Ginny is 42 inches tall. How much taller is Kimo than Ginny?

- Last year 253 people attended the school carnival. This year 397 people attended the carnival. How many more people attended this year?

Missing Addend

- Pam had 27 children's books in her personal library. She gave many of these books away. She kept 10 children's books as these were her children's favorites. How many books did she give away?

- Aleta had 75 cents. She bought bubble gum and got 20 cents change. How much did she spend for the gum?

- Rico had 15 baseball caps. After giving his best friend, Kaukahi, some caps, he had 10 left. How many caps did Rico give Kaukahi?

Suggested questions:

- How are the take away, comparison, and missing addend problems different? How are they modeled with objects or pictures differently?
- How do you know when to regroup and not to regroup?
- Can you subtract from left to right? (e.g., start with tens column for two-digit numbers or start with hundreds column for three-digit numbers). Justify your answer.

- **Extensions**

Students may make up their own problems for the class to solve.

References

Forsten, C. (1992). *Teaching thinking and problem solving in math.* New York: Scholastic Professional Books. pp. 40–41.

Laycock, M. (1977). *Base ten mathematics.* Hayward, CA: Activity Resources Company, Inc. pp. 16–21.

Smith, R. (1986). *Word problems: Grade 2.* Grand Rapids, MI: Instructional Fair, Inc. pp. 6–9.

Standard 8: Whole Number Computation

In grades K–4, the mathematics curriculum should develop whole number computation so that students can—

- model, explain, and develop reasonable proficiency with basic facts and algorithms;
- use a variety of mental computation and estimation techniques;
- use calculators in appropriate computational situations; and
- select and use computation techniques appropriate to specific problems and determine whether the results are reasonable.

Practice in computational algorithms or methods has dominated the mathematics curriculum. With increased use of technology, students should be able to make decisions about when to use certain computations and about the reasonableness of the answers they get. Thus, students should have access to calculators or computers to use along with paper-and-pencil calculations.

The algorithms emphasized in early grades should also be supported by student-invented methods. By allowing alternative algorithms, students are encouraged to think about the techniques rather than routinely applying steps that they do not understand.

Standard 8: Whole Number Computation Activity I

Standard's bullets addressed by activity

- *use a variety of mental computation and estimation techniques*

Technology

- None at the beginning. Calculators can later be used to verify answers after problems are mentally computed.

Description of Activity

- **Group or classroom management practices**

  Students work individually. They can also work in pairs or in small groups to share and discuss their strategies before sharing with the whole class.

Teacher reads the following problem orally and instructs the students to figure out the answer in their heads—no pencil-and-paper computation:

> On my way to work this morning I stopped by the bakery to get something to eat for my usual on-the-run breakfast. I bought a half dozen ham and cheese croissants, one for myself and the rest to share with the office. The croissants were 99¢ apiece (no sales tax). How much did I pay for the croissants?

After mentally calculating their answers, students share their strategies. Some of these may include:

- I rounded each one off to a dollar, multiplied by 6 for the half dozen, and got $6.00. Because I added one penny for each croissant, I took off six cents from the total and got $5.94.

- I multiplied 9 times 6 and got 54¢. Then I multiplied 90 times 6 and got $5.40. I added these together and got $5.94.

- I just multiplied in my head: 9 x 6 = 54. Put the 4 down and carried the 5. Then I multiplied 9 times 6 again and got 54, plus the 5 equals 59. My answer is $5.94.

There may be other strategies that the students invent. The purpose of this activity is to show that there are several ways to arrive at the same answer. Some strategies may be more efficient than others, but the important thing to remember is that the students are thinking. Also, by hearing more efficient strategies from others, they may try these strategies with other problems.

Some questions the teacher might ask:

- (to student with the first response above) Why did you round 99¢ to one dollar?

- We now have three different ways and we all got the same answer. Does anyone else have a different strategy?

- Was there a strategy different from yours that you thought was easier? Why do you think it was easier? Will you use that strategy?

- How useful is mental math in the real world?

- Have any of you used mental math while shopping for food, clothes, or toys? The teacher can also share how he/she uses mental math.

- **Extensions**

Use mental math exercises several times a week, or on a daily basis. Use situations that are meaningful to the students. Students may even make up their own problems and give them to their classmates to solve.

Students should also be encouraged to estimate their answers before computing for exact answers. Calculators can then be used to verify their answers and to see how close their estimates were.

Teachers may also want to discuss situations in which mental math is most efficient to use, when calculators are most efficient to use, and when paper and pencil calculations are most appropriate.

Standard 8: Whole Number Computation Activity II

Standard's bullets addressed by activity

- *model, explain, and develop reasonable proficiency with basic facts and algorithms*
- *use a variety of mental computation and estimation techniques*
- *select and use computation techniques appropriate to specific problems and determine whether the results are reasonable*

Technology

- Calculators can later be used to verify answers after problems are mentally computed.

Description of Activity

- **Group or classroom management practices**

  Students work individually. Students can also work in pairs or in small groups to share and discuss their strategies before sharing with the whole class.

Given situational contexts for basic facts, students use the facts that they already know to arrive at sums of unknown facts. For example:

- Alicia has seven baseball cards. Her best friend gave her nine more. How many baseball cards does she have now?

The students should be able to use derived strategies such as the following:

- I know that 7 + 7 = 14. Because 9 is 2 more than 7, then 7 + 9 has to be 16.
- I know that 8 + 8 = 16. I took one off the 9 to make the 7 an eight. I've changed the problem to 8 + 8.
- I can make the 9 a ten by taking one from the 7. My new problem is 10 + 6 = 16.

Another example:

- Rosa bought 6 packs of pencils. Each pack had 5 pencils. How many pencils did Rosa buy altogether?

Possible strategies:

- I can count by fives for each pack of pencils—5, 10, 15, 20, 25, 30. There are 30 pencils.
- I know that 5 x 5 = 25; one more group of 5 makes 30.

Another example:

- Passing by the candy shelves, Juanita wanted two different kinds of candy. One cost 48¢ and the other cost 23¢ (no sales tax). She has 75¢ with her. Does she have enough money to buy the two kinds of candy?

Possible strategies:

- 48¢ is about 50¢ and 23¢ is about 25¢. Yes, she has enough money. She'll even get change back. Her change will be 4¢.
- I know that 40 + 20 = 60 and 8 + 3 = 11, so 60 + 11 = 71. Both candies will cost 71¢. Yes, she has enough money.

Suggested questions:

- Does anyone else have a different way of solving the problem?

- Was there a strategy different from yours that you thought was easier? Why do you think it was easier? Will you use that strategy with other problems?

- **Extensions**

Give situational problems where the solutions are given. Ask the students if the answers are reasonable. The following is an example:

> Three fourth-grade teachers at Talofofo Elementary School decided to take all their students on a picnic. Mrs. Tenorio spent $26.84 for refreshments. Since the three teachers wanted to share the cost of the picnic, Mrs. Tenorio used her calculator to determine that each teacher should pay her $13.47. Is her answer reasonable? Explain.

After the students figure out that the answer is not reasonable, another extension to this problem would be to ask the students how Mrs. Tenorio got her answer.

Reference

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM. pp. 44-45.

Standard 9: Geometry and Spatial Sense

In grades K–4, the mathematics curriculum should include two- and three-dimensional geometry so that students can—

- describe, model, draw, and classify shapes;
- investigate and predict the results of combining, subdividing, and changing shapes;
- develop spatial sense;
- relate geometric ideas to number and measurement ideas; and
- recognize and appreciate geometry in their world.

Activities in grades K–4 help students develop an intuitive sense about spatial relationships. These relationships emphasize the direction, orientation, and perspectives of objects in space. Further, these concepts promote geometric understandings.

Other investigations and explorations that involve visualizing, drawing, and comparing shapes develop geometric ideas. As students recognize shapes, they are then able to identify properties of the shape. Relationships among shapes develop and finally students make deductions based on their observations.

Standard 9: Geometry and Spatial Sense Activity

Standard's bullets addressed by activity

- *describe, model, draw, and classify shapes*
- *investigate and predict the results of combining, subdividing, and changing shapes*
- *develop spatial sense*

Materials needed

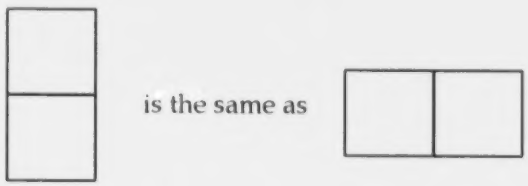
- Square tiles or square pieces of paper

Description of Activity

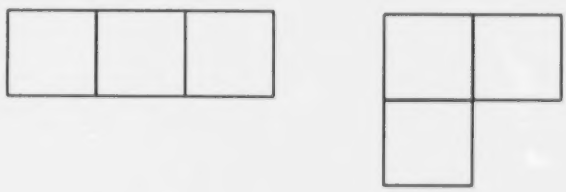
- **Group or classroom management practices**

  Groups of two to four students.

Using two of the squares, ask students how many **different** ways the squares can be put together to form a new shape. As they form new shapes, the sides of the squares must match up. They cannot be put together, for example, only at the corners. Have the class agree that if the shape can be rotated or flipped to form a shape congruent to one already given, then it is not a different shape.



(Students will find only one way to put the squares together.) When they agree on the number of new shapes, now ask them, "What if you started with three squares? How many shapes could you make?" (Students should be able to find two ways to put the squares together.)



Now have students start with four squares. How many new shapes can they find? (Students should find five new shapes.) If you have organized the information on a chart, students may detect a pattern.

Using their observations from the previous trials, ask them to predict, without trying any first, how many new shapes they can find if they start with five squares. Have them write their predictions on their paper. Then, ask them to work in groups to find all of the shapes. Draw the shapes they find on the board or overhead. (Students should find 12 new shapes.)

Once they have found all 12 of the new shapes, now ask them to find the ones that can be folded into an open-top box. Some students may want or need to use large grid paper. The shapes can be drawn on the grid paper, cut out and folded to test for an open box.

- **Extensions**

Ask students to predict the number of new shapes they could create if they started with six squares. After they write their prediction, they should draw all possible shapes. Using their observations from those that could be folded into open-top boxes, students should predict the number of their figures that can be folded into cubes. Now find them. What patterns do they notice? What if there were seven boxes?

Standard 10: Measurement

In grades K–4, the mathematics curriculum should include measurement so that students can—

- understand the attributes of length, capacity, weight, mass, area, volume, time, temperature, and angle;
- develop the process of measuring and concepts related to units of measurement;
- make and use estimates of measurement; and
- make and use measurements in problem and everyday situations.

Measurement is a natural link between mathematics and real-world contexts. Students see measurement used in everyday situations and can understand the relevance to their own lives. As they become proficient with measuring, they will acquire knowledge about what tools to select for measurement tasks and what units would be most appropriate.

Measurement activities should begin with nonstandard units in each measurement attribute before students use standard units. They should explore the effects on measurements when the unit is changed. Most importantly, they should be actually measuring rather than looking at pictures in a textbook. Moreover, measurement can be used as a context to introduce and explore rational numbers (fractions and decimals).

Standard 10: Measurement Activity

Standard's bullets addressed by activity

- *understand the attributes of length*
- *develop the process of measuring and concepts related to units of measurement*
- *make and use estimates of measurement*
- *make and use measurements in problem and everyday situations*

Materials needed

- Recording sheets (student created)

Description of Activity

- **Group or classroom management practices**

  Students work in pairs.

Each pair will estimate and then measure the length of the designated section of the school playground in giant steps. While one partner paces, the other counts. The students switch roles and repeat the activity. The students will record their results on a piece of paper. They will also need to calculate how close their estimates were to the actual number of giant steps. A recording sheet may look like this:

Name	Estimate	No. of Giant Steps	Difference

Students will compare their results with their partner's. The teacher should also have a similar recording sheet on chart paper or on the chalkboard for students to record their measurements as a whole class. They should notice that the number of steps will be different from one student to another, but possibly within a close range.

Suggested questions to ask in the large group discussion:

- Were your estimates close to the actual number of giant steps you took? Why or why not?
- Why are the measurements of giant steps different for all of us? Are there any measurements that are exactly the same? Why is this?
- What would happen if I measured with my giant steps? Will my measurement be close to any of yours? Justify your answer.
- Is "giant steps" a good way to measure the length of the playground? Explain.
- What kind of measurement can we use so that we all come up with the same number? Is coming up with the same number important? Why or why not? (Discuss the importance of standard units of measurement. Integration of language arts is highly encouraged – read storybooks like *How Big Is A Foot?*)
- Estimate how many inches or feet your "giant step" measures.

- **Extensions**

Have students estimate and then measure lengths of certain sections around their home with "baby steps." Suggested sections to measure can include: length of their bedroom, length of their house, length from the front door to the back door, and so on. Students will decide for themselves how to best record their data. Have them estimate how many inches long their "baby step" is. Begin next day's class discussing their findings.

Reference

Corwin, R. & Russell, S. (1990). *Used numbers*. Palo Alto, CA: Dale Seymour Publications. pp. 14-15.

Standard 11: Statistics and Probability

In grades K–4, the mathematics curriculum should include experiences with data analysis and probability so that students can—

- collect, organize, and describe data;
- construct, read, and interpret displays of data;
- formulate and solve problems that involve collecting and analyzing data; and
- explore concepts of chance.

Statistics and probability span across disciplines such as social studies and science. They are present in business and economic contexts and appear in newspapers and trade magazines regularly. Students need the decision-making and prediction skills that statistics and probability develop.

Students should actively explore the collection, organization, description, and interpretation of data. The conclusions they are able to draw from their data collections provide the basis for decisions and predictions. This is a more powerful tool than the memorization of statistical formulas or of computation methods for probability.

Standard 11: Statistics and Probability Activity

Standard's bullets addressed by activity

- *collect, organize, and describe data*
- *construct, read, and interpret displays of data*
- *formulate and solve problems that involve collecting and analyzing data*

Materials needed

- Poster board or chalkboard
- Pictures or drawings of different modes of getting to school (person walking, bus, car, bicycle, taxi, roller blades, boat, and so on)
- Students' pictures or drawings of themselves

Technology

- Computers with software that have graphing capabilities is optional but highly recommended.

Description of Activity

- **Group or classroom management practices**

  Work as a whole class.

Students are told to bring small pictures of themselves or to make drawings of themselves if they don't have a photograph. The teacher has a picture graph on chart paper or on the chalkboard similar to this:

HOW WE GET TO SCHOOL

walk					
bus					
car					
bicycle					
taxi					
boat					
other					

Students are called individually to place their pictures in the appropriate category. When this is done, have the students analyze the graph and generate their own conclusions. The teacher might say, "Take a look at the picture graph we made. What does the graph tell us?" Possible responses may be that no one rides a bike to school or that more students take the bus to school.

Suggested questions:

- How should we record the mode of getting to school for students who get to school in several different ways? For example, sometimes they ride the bus, sometimes they ride in the car, and sometimes they walk.

- How do graphs give us information?

- Can we graph our data using other kinds of graphs? What kinds?

- Can we report our data in another form besides graphs? If so, what and how?

- **Extensions**

Students can work in groups and collect their own data. They decide for themselves how best to collect, record, and report the data. Student-generated surveys like "favorite school subjects" or "favorite types of pizza" are two suggestions. Students can also look at various publications (e.g., newspapers or magazines) and see what types of statistics are reported and how they are reported.

Reference

Shulte, A. (Ed.), (1981). *1981 Yearbook: Teaching statistics and probability.* Reston, VA: NCTM. p. 36.

Standard 12: Fractions and Decimals

In grades K–4, the mathematics curriculum should include fractions and decimals so that students can—

- **develop concepts of fractions, mixed numbers, and decimals;**
- **develop number sense for fractions and decimals;**
- **use models to relate fractions to decimals and to find equivalent fractions;**
- **use models to explore operations on fractions and decimals; and**
- **apply fractions and decimals to problem situations.**

Fractions and decimals provide a context for students to describe observations related to measurement, probability, and statistics. Students have multiple experiences with fractions outside the school classroom and can use them to model fractions and decimals in multiple ways.

Usually, when one describes experiences with fractions in school, the predominant features are the algorithms that are needed to perform computations. However, the K–4 curriculum should focus on a conceptual understanding of how large a fraction is and on developing a language related to fractions and decimals. These skills are an important foundation for later work in fractions and decimals.

Students can use benchmark fractions (0, $\frac{1}{2}$, and 1) to describe the size of fractions and decimals that cluster around them. The benchmarks can then be used to predict the size of a sum or difference **before** students learn the algorithms.

Standard 12: Fractions and Decimals Activity

Standard's bullets addressed by activity

- *develop concepts of fractions*
- *develop number sense for fractions*

Materials needed

- Four strips of 3" x 18" construction paper of different colors
- Scissors

Description of Activity

- **Group or classroom management practices**

  Students' desks should be grouped in fours to form a table-like pattern.

Introduce the activity by holding up a piece of gum. If I open this gum and put it all in my mouth, how much of the gum will I have in my mouth? [one whole]

If I share the gum equally with Momi, how much will each of us get? [one-half]

If I share the gum equally with Momi and Keoni, how much will each of us get? [one-third]

How many people would get gum if I divided the gum so that each piece is one-fourth of the whole?

How many parts would I have to divide it into so that each person in the room would get some?

The teacher should prepare strips of construction paper 3" x 18" from several different colors. Have students choose seven strips. One strip should be designated as the "WHOLE." Have students measure the strips by placing them on top of each other and discovering that they are all the same size as the "ONE WHOLE." Talk about the activity which will divide the strips into fractional parts.

Each student will label one strip as "one whole." Another strip will be folded to represent "one-half" of the whole (3" x 18") strip. Do the same for thirds, fourths, eighths, and so on.

Suggested questions:

- How many different ways are there to show "one-half" of this "whole" strip?

- Using the 3" x 18" strip, let's assume that several students folded their halves so that their "one-half" measures 3" x 9" and another group's "one-half" measures 1 1/2" x 18". Taking an example of each "half," ask if both "halves" are the same or equal. How are they the same? Can you prove it? (Do the same with the other fractional pieces.)

- Comparing unit fractions, have students justify why one fraction is larger or smaller than the other. For example: Is "one-half" bigger than "one-third"?

How do you know? Shouldn't "one-third" be bigger than "one-half" as "three" is bigger than "two"?

- **Extensions**

Explore concepts such as these:

- Is "one-half" always equal to "one-half"? Justify your answer.
- Is "one-half" always larger than "one-fourth"? Explain your answer.
- If you have "one-half" what other parts could you use to make "one whole"?
- If you have "five-eighths" and you want to have "one whole", how many more eighths would you need?
- Which is more— "three-eighths" or "two-fourths"? How do you know?

Reference

Stenmark, J., Thompson, R., & Cossey, R. (1986). *Family math.* Berkeley, CA: Lawrence Hall of Science, University of California-Berkley. pp. 120-123.

Standard 13: Patterns and Relationships

In grades K–4, the mathematics curriculum should include the study of patterns and relationships so that students can—

- recognize, describe, extend, and create a wide variety of patterns;
- represent and describe mathematical relationships; and
- explore the use of variables and open sentences to express relationships.

Describing patterns helps students make generalizations about mathematics and the world they live in. Patterns can be found in numerical, geometrical, measurement, or abstract contexts and there may be multiple generalizations for the same pattern.

Standard 13: Patterns and Relationships Activity

Standard's bullets addressed by activity

- *recognize, describe, extend, and create a wide variety of patterns*
- *represent and describe mathematical relationships*

Materials needed

- Graph paper, colored tiles, small squares, or something similar

Description of Activity

- **Group or classroom management practices**

  Students work individually, in pairs or in small groups of three to four members.

Introductory Activity: Samoan sasa (using auditory sounds)

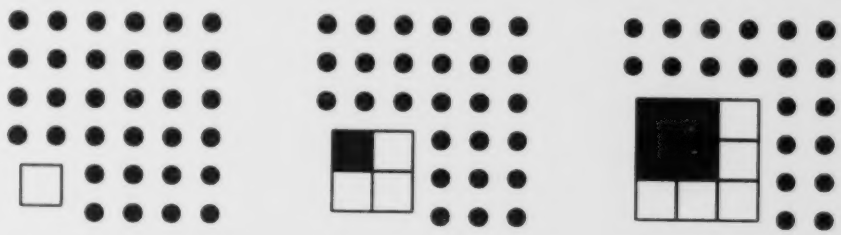
- clap (pati) – with open hands
- clap (po) – with semi-open hands

Teacher demonstrates while students listen. Go through each part separately and have students repeat it.

- 3 pati, 3 po, 1 pati
- 4 pati, 4 po, 1 pati, 1 po, 2 pati

Have students make their own patterns with the clapping or body movements such as sitting, jumping, hopping, and so on.

Follow-up Activity: Using the illustrations below find as many patterns as you can.



1
1
1 X 1

1 + 3
4
2 X 2

1 + 3 + 5
9
3 X 3

Suggested questions:

- What patterns can you see with the squares?
- What would the next set of squares look like? Draw or build what the next square would look like.

- **Extensions**

Use the calculator to explore number patterns. The calendar is another good place to look for patterns.

Reference

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM. p. 61.

GRADES 5 - 8
STANDARDS

Standard 1: Mathematics as Problem Solving

In grades 5–8, the mathematics curriculum should include numerous and varied experiences with problem solving as a method of inquiry and application so that students can—

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from situations within and outside mathematics;
- develop and apply a variety of strategies to solve problems, with emphasis on multi-step and nonroutine problems;
- verify and interpret results with respect to the original problem situation;
- generalize solutions and strategies to new problem situations; and
- acquire confidence in using mathematics meaningfully.

Mathematics as problem solving implies that problem solving is an integral part of the mathematics class. For many years problem-solving tasks have been given as peripheral to learning mathematics. We now move to viewing problem solving as a way of teaching and learning rather than as extra problems.

Problem solving can be thought of in at least two ways. First, problem solving represents a method of instruction where teachers ask process-type questions that encourage students to explore. These questions may include: Can you solve the problem in another way? Is there another solution? How can you justify your solution process? How does this problem relate to others that you've solved? Process or problem-solving questions require extended answers and motivate students to look at the mathematics in a deeper fashion.

Problem solving also suggests the type of problem that students solve. These standards advocate using problems that require an extended time period to solve, are solved with a variety of problem-solving strategies (working backwards, make a table, guess-and-test, and so on), and can be extended to probe a concept or idea even further. These problems do not lend themselves to solutions by applying computational methods. They promote higher-level thinking and creative solution techniques.

Standard 1: Problem-solving Activity

Standard's bullets addressed by activity

- *use problem-solving approaches to investigate and understand mathematical content*
- *formulate problems from situations within and outside mathematics*
- *develop and apply a variety of strategies to solve problems, with emphasis on multi-step and nonroutine problems*
- *verify and interpret results with respect to the original problem situation*
- *acquire confidence in using mathematics meaningfully*

Materials Needed

- Cups, gallon containers, buckets, or other containers

Technology

- Calculator

Description of Activity

- **Group or classroom management practices**

  Groups of two to four students

The Samson family has gone on a two-week vacation. Unfortunately, they left a faucet dripping in their bathroom. The faucet drips 2 times per second. How many times does the faucet drip every minute, hour, day? After one day, how much water was wasted?

- **Extensions**

How much water (in gallons) is a drip? Explain your method used to determine your solution. Are all drips the same size? What factors affect drip size? Can a faucet drip two gallons per hour? Explain your reasoning.

Reference

Scott, Foresman and Company (1991). Exploring mathematics: Problem solving and critical thinking.

Standard 2: Mathematics as Communication

In grades 5–8, the study of mathematics should include opportunities to communicate so that students can—

- model situations using oral, written, concrete, pictorial, graphical, and algebraic methods;
- reflect on and clarify their own thinking about mathematical ideas and situations;
- develop common understandings of mathematical ideas, including the role of definitions;
- use the skills of reading, listening, and viewing to interpret and evaluate mathematical ideas;
- discuss mathematical ideas and make conjectures and convincing arguments; and
- appreciate the value of mathematical notation and its role in the development of mathematical ideas.

Communication includes at least five instructional methods: 1) writing, 2) speaking, 3) reading, 4) listening, and 5) constructing or modeling. Writing can involve responses to journal prompts, problems created by students, and descriptions of problem solutions. Speaking is that interaction done by the students as they explain their thinking to other students and the teacher. It is important to note that it is the **students** who are talking and not just the teacher. Reading refers to students reading each other's work or problems or using other resources. Listening allows students an opportunity to critically analyze other students' ideas. Finally, as students construct shapes or model concepts with concrete materials, they communicate ideas about a mathematical topic.

Standard 2: Communication Activity

Standard's bullets addressed by activity

- *model situations using oral, pictorial, graphical, and algebraic methods*
- *reflect on and clarify their own thinking about mathematical ideas and situations*

Materials needed

- Graph paper might be useful.

Description of Activity

- **Group or classroom management practices**

  Teacher discretion

Suppose your best friend asks you to explain slope. Write about what you would say or show this person. What would be the best way to explain slope to your friend? How can you be sure you were understood?

- **Extensions**

Draw a picture that best explains your idea of slope. How would you illustrate "no slope"? What if the line had a slope of 0? How is a slope of 0 alike or different from "no slope"? What is the largest slope a line can have? Why? What is the smallest slope a line can have? Why?

Standard 3: Mathematics as Reasoning

In grades 5–8, reasoning shall permeate the mathematics curriculum so that students can—

- recognize and apply deductive and inductive reasoning
- understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs;
- make and evaluate mathematical conjectures and arguments;
- validate their own thinking; and
- appreciate the pervasive use and power of reasoning as a part of mathematics.

Critical thinking is the heart of mathematics. In fact, mathematical problems cannot be solved without reasoning the solution process and deciding if the answer is logical. This standard does not refer to the formal reasoning process but it should portray the conjecturing, validating, refuting, and analyzing processes that are used to make sense of mathematics. Students can use various means to communicate their reasoning such as oral discussion, the construction of models, and written explanations.

Standard 3: Reasoning Activity

Standard's bullets addressed by activity

- *make and evaluate mathematical conjectures and arguments*
- *validate their own thinking*

Materials needed

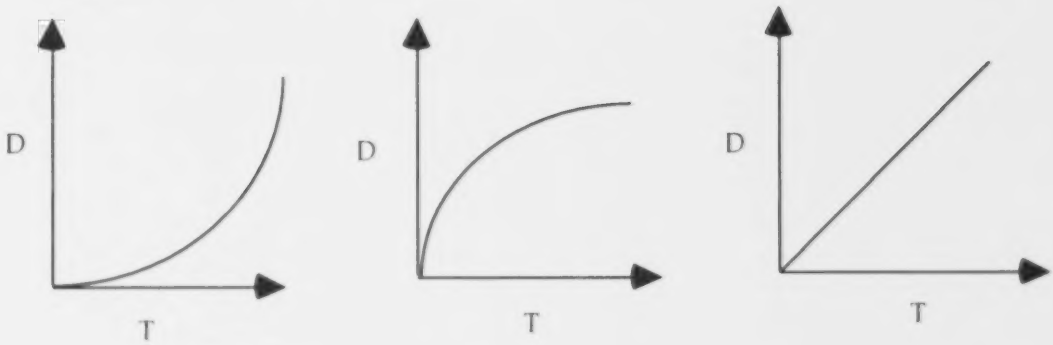
- Graph paper

Description of Activity

- **Group or classroom management practices**

  Students can work alone or in groups.

D=DISTANCE, T=TIME



Which of these three graphs shows the distance of the cyclist from the starting line of a race where she travels over a large hill at the beginning of the race?

Describe a real-world scenario for the remaining two graphs.

- **Extensions**

Suppose you had to walk home from school. About half way home, you realized that you forgot your books and had to return to school to get them. You then had to jog from school (about twice as fast as walking) to make up the lost time. What would a graph of your trip look like? Discuss the changing slope of your graph.

Note to the teacher: *Encourage some students to switch D and T. Compare graphs with other groups.*

Reference

Scott, Foresman & Company, (1991). Exploring mathematics: Problem solving and critical thinking.

Standard 4: Mathematical Connections

In grades 5–8, the mathematics curriculum should include the investigation of mathematical connections so that students can—

- see mathematics as an integrated whole;
- explore problems and describe results using graphical, numerical, physical, algebraic, and verbal mathematical models or representations;
- use a mathematical idea to further their understanding of other mathematical ideas;
- apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as art, music, psychology, science, and business; and
- value the role of mathematics in our culture and society.

Mathematics is often thought of as separate topics that are not intertwined but are studied in isolation. Connecting the topics helps students form a more cohesive knowledge base. This eliminates the memorization of facts or procedures that appear to have little meaning.

Connections also relate directly to how students learn mathematics. Students who make connections usually explore a mathematical topic for a period of days and then bridge ideas that are represented in concrete form to a more abstract one. Additionally, they are able to discern similarities and differences among ideas and procedures.

Standard 4: Connections Activity

Standard's bullets addressed by activity

- *explore problems and describe results using graphical, numerical, physical, algebraic, and verbal models or representations*
- *use a mathematical idea to further their understanding of other mathematical ideas*

Description of Activity

- **Group or classroom management practices**

  Cooperative/collaborative learning groups with three to four students per group

The local art gallery is getting ready for an upcoming art exhibit. They would like to display seven particular paintings on one wall $8\frac{1}{2}$ feet high by $12\frac{1}{2}$ feet long. If the dimensions of the seven paintings are as follows, design how they could place the paintings so that they are all displayed and there is at least 2 inches between each of them.

Numbers of paintings	Height (in feet)	Length (in feet)
1	2	5
3	3	4
2	4	3
1	5	4

Describe your method for designing the wall.

Standard 5: Number and Number Relationships

In grades 5–8, the mathematics curriculum should include the continued development of number and number relationships so that students can—

- understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, and scientific notation) in real-world and mathematical problem situations;
- develop number sense for whole numbers, fractions, decimals, integers, and rational numbers;
- understand and apply ratios, proportions, and percents in a wide variety of situations;
- investigate relationships among fractions, decimals, and percents; and
- represent numerical relationships in one- and two-dimensional graphs.

Students learn to work a variety of types of numbers, such as fractions, decimals, percents, ratios, and integers. In a traditional curriculum, students would encounter each of these in isolated settings or would spend time learning to write a decimal as a fraction and so on. Students should be able to represent numbers in a variety of ways and their chosen representations should be appropriate to the context. Thus, number problems involving number representations should not be done without a contextual situation in which students must make decisions.

Standard 5: Number and Number Relationships Activity

Standard's bullets addressed by activity

- *understand and apply ratios, proportions and percents in a wide variety of situations*
- *investigate relationships among fractions, decimals and percents*

Materials needed

- Container (coconut shell) to hold small portions of beans
- Three types of dried beans: kidney, green peas, white navy

Description of Activity

- **Group or classroom management practices**

  Cooperative/collaborative learning groups with three to four students per group

Suppose you are to make the following salads. Each salad contains all three types of beans. For each salad determine how many of each of the three types of beans are needed. Record your results in the table.

Salad 1
This salad contains:
2 kidney beans
Twice as many green peas as kidney beans
10 beans in all

Salad 2
This salad contains:
4 white navy beans
1/2 as many green peas as white navy beans
10 beans in all

Salad 3
This salad contains:
Kidney beans make up 1/2 this salad
The salad has exactly 2 green peas
The number of kidney beans is double the number of green peas

Salad 4
This salad contains:
The same number of green peas as kidney beans
3 more white navy beans than green peas
A total of 18 beans

Salad 5
This salad contains:
12 beans.
1/2 of the beans are green
1/4 of the salad are kidney beans

Salad 6
This salad contains:
At least 12 beans
One more green pea than white navy beans
One more white navy bean than kidney beans

Salad 7
This salad contains:
3 times as many green peas as kidney beans
One more white navy bean than green peas
8 beans in all

Salad 8
This salad contains:
An equal number of green peas and white navy beans
5 more kidney beans than green peas
No more than 20 beans

SALAD	RED KIDNEY	GREEN PEAS	WHITE NAVY	TOTAL BEANS
SALAD 1				
SALAD 2				
SALAD 3				
SALAD 4				
SALAD 5				
SALAD 6				
SALAD 7				
SALAD 8				

- **Extension**

Make up a different salad. Write instructions for someone else to make your salad.

Reference

Stenmark, J. K., Thompson, V., and Cossey, R. (1986). *Family math.* Berkeley, CA: Lawrence Hall of Science. pp. 134-135.

Standard 6: Number Systems and Number Theory

In grades 5–8, the mathematics curriculum should include the study of number systems and number theory so that students can—

- understand and appreciate the need for numbers beyond the whole numbers;
- develop and use order relations for whole numbers, fractions, decimals, integers, and rational numbers;
- extend their understanding of whole number operations to fractions, decimals, integers, and rational numbers;
- understand how the basic arithmetic operations are related to one another; and
- develop and apply number theory concepts (e.g., primes, factors, and multiples) in real-world and mathematical problem situations.

This standard emphasizes the structure of mathematics through relationships of operations and number ideas. The intent is not that students would memorize more procedures but that they would be encouraged to create their own algorithms through explorations and the identification of relationships or patterns.

Standard 6: Number and Number Theory Activity

Standard's bullets addressed by activity

- *develop and use order relations for whole numbers, fractions, decimals, integers, and rational numbers*
- *extend their understanding of whole number operations to fractions, decimals, integers, and rational numbers*
- *understand how the basic arithmetic operations are related to one another*

Materials needed

- Maze Playing Board (included)
- Markers (a coin, cardboard square or circle, or button)

Technology

- **Calculators**

Description of Activity

- **Group or classroom management practices**

  Individuals in whole class or any other configuration

Each individual has his/her own playing board and marker on the start number, 100. Each chooses a route to take from start to finish performing the indicated operation on the calculator as the marker is moved. Rules are that the student can move the marker following any horizontal or down segments. Moves *cannot* be made along an upward segment. Students compete for the highest score at Finish. After the first round, the whole class discusses strategies discovered by students to compute the highest available score (6332). Let the students try several times to find the path that would lead to the high score.

- **Extension**

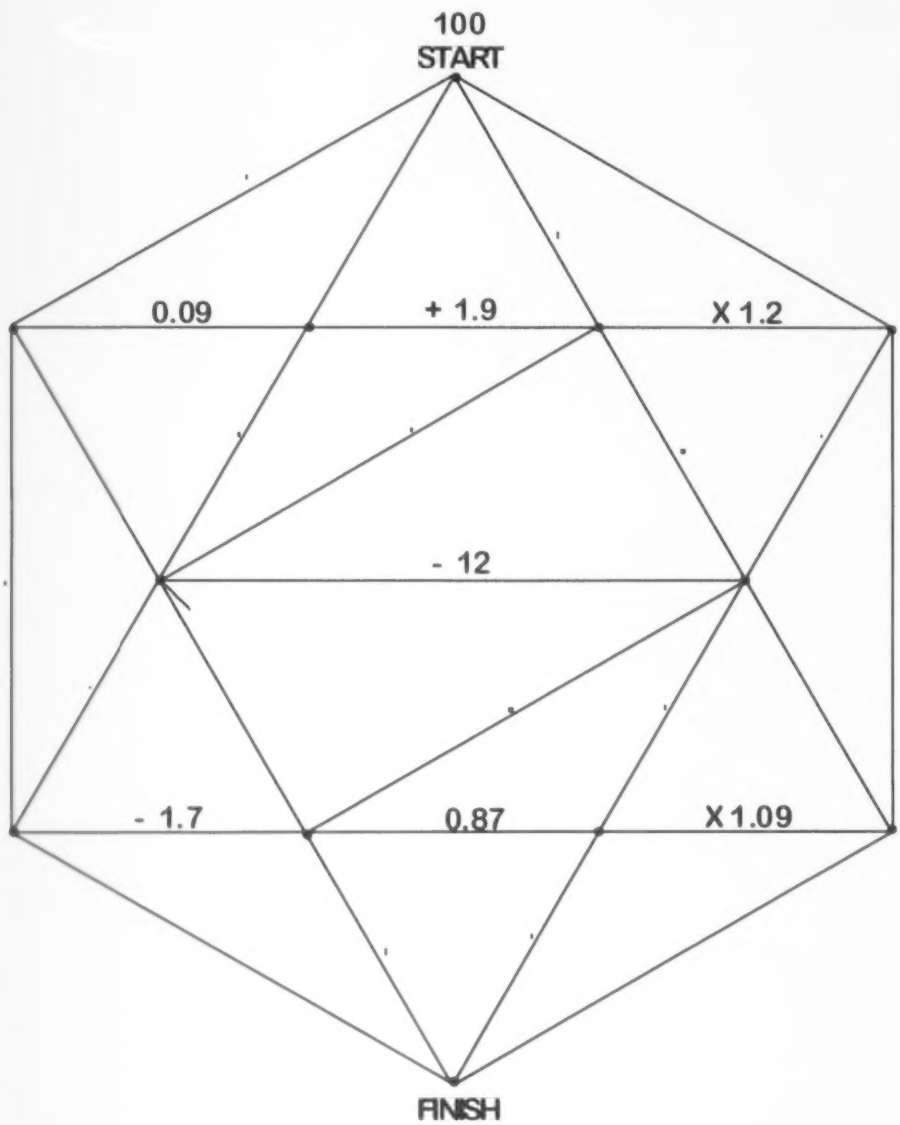
The game can be changed so that the students try for the lowest score possible. Students can also make their own rules to devise a new game.

Reference

Morris, J., (1981). *How to develop problem solving using a calculator.* Reston, VA: National Council of Teachers of Mathematics. pp. 26–27.

Maze

Playing Board



Morris, J., (1981). *How to develop problem solving using a calculator.* Reston, VA: National Council of Teachers of Mathematics. pp. 26–27.

Standard 7: Computation and Estimation

In grades 5–8, the mathematics curriculum should develop the concepts underlying computation and estimation in various contexts so that students can—

- compute with whole numbers, fractions, decimals, integers, and rational numbers;
- develop, analyze, and explain procedures for computation and techniques for estimation;
- develop, analyze, and explain methods for solving proportions;
- select and use an appropriate method for computing from among mental arithmetic, paper-and-pencil, calculator, and computer methods;
- use computation, estimation, and proportions to solve problems; and
- use estimation to check the reasonableness of results.

Many students have experienced mathematics as a series of facts and procedures that must be memorized. These facts and procedures are generally presented as isolated facts without a situational context. It is difficult for students to form a coherent body of knowledge when information is presented in this fashion.

This standard promotes learning about numbers through relationships among other numbers, operations, and explorations that provide opportunities for observations about number ideas. Thus, ideas about computational methods can help students develop their own algorithms or techniques. It is, however, not just important to develop methods of computing but students should also learn how to select appropriate tools for computation, such as calculators, paper-and-pencil, and estimation. Sometimes students automatically use paper-and-pencil for computations that can be easily done as mental mathematics or that can be estimated to give a more appropriate answer. It is not the purpose of this standard to promote the accumulation of more memorized facts.

Standard 7: Computation and Estimation Activity

Standard's bullets addressed by activity

- *develop, analyze, and explain procedures for computation and techniques for estimation*
- *use estimation to check the reasonableness of results*

Materials needed

- Four-Decimals-in-a-Row activity sheet (included)

Technology

- One calculator

Description of Activity

- **Group or classroom management practices**

  Whole class, divided into two teams

Choose one member of the class to be the "calculator." Provide that student with a calculator. Divide the rest of the class into two teams. To begin, display the grid and the factor boards below on the overhead or pass the sheets to everyone in the class. In turn, each team chooses two factors, one from the circular factor board and one from the square factor board. If the product of those numbers is displayed on the grid, the team captures that cell. The first team to capture four cells in a row (vertically, horizontally, or diagonally) is the winning team.

- **Extensions**

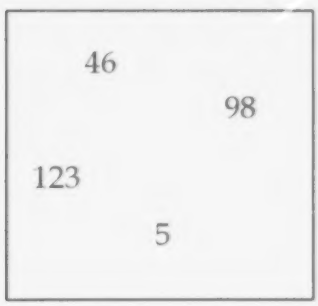
Ask students to create their own boards including the factor boards. Use them in another class competition.

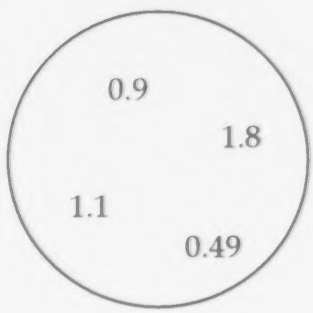
Reference

Reys, B., Barger, R., Dougherty, B., Lembke, L., Parnas, A., Sturdevant, R., Bruckheimer, M., Hope, J., Markovits, Z., Weber, M., (1991). *Developing number sense in the middle grades.* Reston, VA: NCTM. p. 37.

Four-Decimals-in-a-Row

221.4	**88.2**	**82.8**	**110.7**
107.8	**9**	**60.27**	**135.3**
2.45	**176.4**	**41.4**	**48.02**
4.5	**50.6**	**5.5**	**22.54**





Standard 8: Patterns and Functions

In grades 5–8, the mathematics curriculum should include explorations of patterns and functions so that students can—

- describe, extend, analyze, and create a wide variety of patterns;
- describe and represent relationships with tables, graphs, and rules;
- analyze functional relationships to explain how a change in one quantity results in a change in another;
- use patterns and functions to represent and solve problems; and
- use and identify patterns that occur naturally in nature.

Students explored patterns in K–4 in a variety of contexts. Now, in grades 5–8, students can use their experiences to explore introductory notions about functions. There should be a variety of contexts, but the emphasis now shifts to functions and other ways of describing a pattern such as tables and graphs.

Standard 8: Patterns and Functions Activity

Standard's bullets addressed by activity

- *describe, extend, analyze, and create a wide variety of patterns*
- *describe and represent relationships with tables, graphs, and rules*
- *analyze functional relationships to explain how a change in one quantity results in a change in another*

Technology

- Calculators optional

Description of Activity

- **Group or classroom management practices**

  Teams of four students

Play: "Break My Rule"

Work in teams of four. Each team creates a rule and writes a sequence that follows the rule. For example: {1, 6, 11, ...}. The object of the game is to guess another team's rule or prove the existence of another rule. There are several rules that may be given for the example rule. For instance, students may say that the rules are:

- There is a one in the ones place and a six in the ones place in every other number.
- It's skip counting by fives starting with one.
- Even, odd, even, odd.
- You add five each time, starting with one.

- **Extensions**

Students should create other patterns that involve items other than numbers. For example, they could use geometric shapes, music, or physical movements. Follow the same format as in the previous activity.

Students could be given a rule and asked to create a sequence that they believe illustrates the rule.

Standard 9: Algebra

In grades 5–8, the mathematics curriculum should include explorations of algebraic concepts and processes so that students can—

- understand the concepts of variable, expression, and equation;
- represent situations and number patterns with tables, graphs, verbal generalizations, and equations and explore the interrelationships of these representations;
- analyze tables and graphs to identify properties and relationships;
- develop confidence in solving linear equations using concrete, informal, and formal methods;
- investigate inequalities and nonlinear equations informally;
- apply algebraic methods to solve a variety of real-world and mathematical problems; and
- continue to explore patterns and relationships.

(Note: Linear equations would include equations that have a line as their solution—two variables.)

Algebra in the middle grades is a time for transition from arithmetic to more generalizations that can be expressed in multiple ways. The emphasis of this standard is on finding patterns that can be expressed with a table, a verbal description, a graph, or an equation. The formal methods used to solve equations or perform other algebraic manipulations are not presented. Rather, students use concrete methods to solve problems but describe their procedures in general terms.

Algebra has typically been thought of as very abstract, with the outcome being that students can solve equations in a very prescribed step-by-step way. The notion of what algebra represents is changing. Algebra is finding patterns and then describing those in a variety of ways.

Standard 9: Algebra Activity

Standard's bullets addressed by activity

- *represent situations and number patterns with tables, graphs, verbal generalizations and equations and explore the interrelationships of these representations*
- *analyze tables and graphs to identify properties and relationships*
- *apply algebraic methods to solve a variety of real-world and mathematical problems*
- *continue to explore patterns and relationships*

Technology

- One calculator per student desired, graphing calculator if possible

Description of Activity

- **Group or classroom management practices**

  Cooperative/collaborative groups of three to four students

For every field goal a team scores in a basketball game, the team is awarded two points. What would be the team's score after five field goals? after six goals? Make a table to represent the number of points a team would receive for 5 through 12 goals. Could you predict the score after *n* goals?

List three things you notice about the table. Make a graph with the information in the table. List three things you notice about the graph.

Change the rules and award a team three points for each goal. What happens to the table? What changes occurred in the graph?

Can you think of another situation where a table and graph could be made similar to these? Describe the situation and sketch or draw the graph.

This activity introduces the notion of functions, linear relationships and slope.

Standard 10: Probability and Statistics

In grades 5–8, the mathematics curriculum should include exploration of probability and statistics in real-world situations so that students can—

- model situations by devising and carrying out experiments or simulations to determine probabilities;
- model situations by constructing a sample space to determine probabilities;
- appreciate the power of using a probability model by comparing experimental results with mathematical expectations;
- make predictions that are based on experimental or theoretical probabilities;
- develop an appreciation for the pervasive use of probability in the real world;
- systematically collect, organize, and describe data;
- construct, read, and interpret tables, charts, and graphs;
- make inferences and convincing arguments that are based on data analysis;
- evaluate arguments that are based on data analysis; and
- develop an appreciation for statistical methods as powerful means for decision making.

Statistics and probability span across disciplines, such as social studies and science. They are present in business and economic contexts and regularly appear in newspapers and trade magazines. Students need the decision-making and prediction skills that statistics and probability develop.

Students should actively explore the collection, organization, description, and interpretation of data. The conclusions they are able to draw from their data collections provide the basis for decisions and predictions. This is a more powerful tool than the memorization of statistical formulas or of computation methods used to find probability.

Standard 10: Probability and Statistics Activity

Standard's bullets addressed by activity

- *model situations by devising and carrying out experiments or simulations to determine probabilities*
- *model situations by constructing a sample space to determine probabilities*
- *appreciate the power of using a probability model by comparing experimental results with mathematical expectations*
- *systematically collect, organize, and describe data*
- *construct, read, and interpret tables, charts, and graphs*
- *make inferences and convincing arguments that are based on data analysis*
- *evaluate arguments that are based on data analysis*

Materials needed

- Multi-sided dice (six, eight, ten, twenty sides, and so on)

Description of Activity

- **Group or classroom management practices**

  Small groups of three to four students are recommended.

Determine the probability of obtaining a product greater than ten when two dice are rolled. Students could model the problem by constructing a sample space to determine the true (mathematical) probability and compare their results with estimated probabilities found with empirical data (computer generated, human simulations, random number tables, and so on).

- **Extensions**

This problem could be extended by changing the restriction(s) on the event. For example: Determine the probability of obtaining an ***odd product*** when ***three ten-sided*** dice are rolled. Furthermore, a relationship could be found between the number of outcomes in a sample space and the number of dice sides (two six-sided dice produce 6 x 6 = 36 outcomes in a sample space) and hence, an introduction to combinatorics theory is possible.

Reference

National Council of Teachers of Mathematics, (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

Standard 11: Geometry

In grades 5–8, the mathematics curriculum should include the study of the geometry of one, two, and three dimensions in a variety of situations so that students can—

- **identify, describe, compare, and classify geometric figures;**
- **visualize and represent geometric figures as they continue to develop spatial sense and explore spatial relations;**
- **explore transformations of geometric figures;**
- **represent and solve problems using geometric models;**
- **understand and apply geometric properties and relationships; and**
- **develop an appreciation of geometry as a means of describing the physical world.**

Geometry in middle grades should be approached through informal and formal explorations that focus on constructing, drawing, measuring, visualizing, comparing, transforming, and classifying geometric figures. Students will form more complete definitions and the relationships they observe will be more meaningful. They should be encouraged to make deductions and inductions but this is not a formalized procedure of proof work nor should students be asked to memorize a series of characteristics or properties of polygons.

Standard 11: Geometry Activity

Standard's bullets addressed by activity

- *describe and compare geometric figures*
- *understand and apply geometric properties and relationships*

Materials needed

- **Four stations are to be set up with circles to measure (waste basket, tuna can, plate disk). Each station must also have a tape measure in centimeters and a calculator.**
- **Four different colored sets of slips of paper. Each set has a slip with either m, r, s or c on it to determine specific roles within a group - m for measurer, r for recorder, s for statistician, c for checker.**

Technology

- Four calculators minimum—one for each station.

Description of Activity

- **Group or classroom management practices**

  Students work in small groups of three to four students.

Present the following example from history:

> In 1376 BC., the reigning Pharaoh of Egypt died. Ramses became the Pharaoh. Ramses was a fair Pharaoh, but desired a luxurious life, therefore he needed many taxes. In order to determine equal taxes, he had his 'rope stretchers' divide the land into equal parts for each man. The rope stretchers created a formula for the area of a rectangle. When the Nile River flooded, and the need was created, the 'rope stretchers' discovered a formula for the area of a triangle. The one large problem they had was creating a formula for the area of a circular region. They thought a key factor would be the relationship between the diameter of a circle and its circumference. If this relationship could be determined, it would make their work much easier as they sought to determine the measure of large circular areas. Finding this relationship is your task!

The students are divided in groups of four by drawing slips of colored paper — red, blue, yellow, and so on. Each color determines a group. Each slip has **m**, **r**, **s**, or **c** to determine their task—(a) a measurer: to measure diameter and circumference of the items,
(b) a recorder: to record the measurements, (c) a statistician: to find two ratios: diameter/circumference, circumference/diameter, then find the mean average of each ratio, (d) a checker/praiser: to make sure each person does his/her job correctly, then praise accordingly. Students rotate the various jobs as they move to different stations, measuring, recording, and finding ratios. Using calculators, groups find and record ratios among their measurements. Averages are then found and recorded on the board for comparison with other groups.

During the last ten minutes of class, the entire class gets together to discuss results. Is there a relationship between the similar ratios? Do you suppose this is true in all circles? What equation can we form showing the relationship of circumference and diameter? How can this relationship be useful? What is a circle? What parts of a circle do you know? Illustrate points "in," "on," "outside" a circle.

- **Extensions**

Find four circular items at home and repeat the experiment.

At the beginning of class the next day, students have five minutes to write in their journal comments regarding this assignment, their expectations, the result, how their group functioned and so on.

Reference

Serra, M. (1993). *Discovering geometry*. Berkeley, CA: Key Curriculum Press.

Standard 12: Measurement

In grades 5–8, the mathematics curriculum should include extensive concrete experiences using measurement so that students can—

- extend their understanding of the process of measurement;
- estimate, make, and use measurements to describe and compare phenomena;
- select appropriate units and tools to measure to the degree of accuracy required in a particular situation;
- understand the structure and use of systems of measurements;
- extend their understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight and mass;
- develop the concepts of rates and other derived and indirect measurements; and
- develop formulas and procedures for determining measures to solve problems.

Real world connections are the focal point of middle grades measurement. Students should focus on measurement in problem contexts rather than performing conversions on measurements. Activities should include the selection of appropriate tools and units for the situation.

Even though there are formulas for formal measurements such as perimeter and area, students should not rely on them. Instead they should explore other techniques that give a meaningful look at why formulas work or were developed. Estimation and the judgment about reasonable answers are both crucial to the middle school curriculum.

Standard 12: Measurement Activity

Standard's bullets addressed by activity

- *extend their understanding of the process of measurement*
- *estimate, make, and use measurements to describe and compare phenomena*
- *select appropriate units and tools to measure to the degree of accuracy required in a particular situation*
- *understand the structure and use of systems of measurements*

- *extend their understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight and mass*

Materials needed

- Resource materials on Japan (Mt. Fuji, specifically)

Technology

- Calculators needed here!

Description of Activity

- **Group or classroom management practices**

  Teacher discretion

How much dirt is necessary to fill a dump truck? Draw a diagram that models the problem and give your answer in inches^3, feet^3, and yards^3. **How much would the dirt weigh?**

- **Extensions**

How long would it take to remove Mt. Fuji with dump trucks? What is Mt. Fuji's mass? Assume there is a continuous filling of dump trucks with loads taken away every 10 minutes. Explain your reasoning.

GRADES 9 - 12
STANDARDS

Standard 1: Mathematics as Problem Solving

In grades 9–12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can—

- use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;
- apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems from situations within and outside mathematics; and
- apply the process of mathematical modeling to real-world problem situations.

Mathematics as problem solving implies that problem solving is an integral part of the mathematics class. For many years problem-solving tasks have been given as peripheral to learning mathematics. We now move to viewing problem solving as a way of teaching and learning rather than as extra problems.

Problem solving can be thought of in at least two ways. First, problem solving represents a method of instruction where teachers ask process-type questions that encourage students to explore. These questions may include: Can you solve the problem in another way? Is there another solution? How can you justify your solution process? How does this problem relate to others that you've solved? Process or problem-solving questions require extended answers and motivate students to look at the mathematics in a deeper fashion.

Problem solving also suggests the type of problem that students solve. These standards advocate using problems that require an extended time period to solve, are solved with a variety of problem-solving strategies (working backwards, make a table, guess-and-test, and so on), and can be extended to probe a concept or idea even further. These problems do not lend themselves to solutions by applying computational methods. They promote higher-level thinking and creative solution techniques.

Standard 1: Problem-solving Activity

Standard's bullets addressed by activity

- *use with increasing confidence, problem-solving approaches to investigate and understand mathematical content*
- *apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics*
- *recognize and formulate problems from situations within and outside mathematics*
- *apply the process of mathematical modeling to real-world problem situations*

Description of Activity

- **Group or classroom management practices**

  Cooperative/collaborative learning groups with three to four students per group

Present the groups with the following problem.

Suppose that you have an unlimited supply of 5-cent stamps and 11-cent stamps. You can make exact postage of 21 cents with two 5-cent stamps and one 11-cent stamp. However, you *cannot* make exact postage of 23 cents by using only 5-cent stamps and 11-cent stamps. What is the greatest amount of exact postage you *cannot* make by using these stamps.

Have students work collaboratively to solve the problem. Groups should write a summary on solution processes, patterns that developed, and generalizations they might have.

- **Extension**

What if the stamp values were 3¢ and 8¢ instead of 5¢ and 11¢? What patterns do you notice that would help predict for any two values of stamps?

Reference

Rachlin, S., Matsumoto, A., and Wada, L . (1992). *Algebra I: A process approach.* Honolulu, HI: Curriculum Research & Development Group. p 32.

Standard 2: Mathematics as Communication

In grades 9–12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can—

- reflect upon and clarify their thinking about mathematical ideas and relationships;
- formulate mathematical definitions and express generalizations discovered through investigations;
- express mathematical ideas orally and in writing;
- read written presentations of mathematics with understanding;
- ask clarifying and extending questions related to mathematics they have read or heard about; and
- appreciate the economy, power, and elegance of notation and its role in the development of mathematical ideas.

Communication includes at least five instructional methods: 1) writing, 2) speaking, 3) reading, 4) listening, and 5) constructing or modeling. Writing can involve responses to journal prompts, problems created by students, and descriptions of problem solutions. Speaking is that interaction done by the students as they explain their thinking to other students and the teacher. It is important to note that it is the **students** who are talking and not just the teacher. Reading refers to students reading each others' work or problems or using other resources. Listening allows students an opportunity to critically analyze other students' ideas. Finally, as students construct shapes or model concepts with concrete materials, they communicate ideas about a mathematical topic.

Standard 2: Communication Activity

Standard's bullets addressed by activity

- *express mathematical ideas orally and in writing*

Description of Activity

- **Group or classroom management practices**

Teacher discretion

Have students write an essay on how they solve a quadratic equation. To ensure the students describe more than one method, provide the following prompt:

> I can solve a quadratic equation by _____, _____, _____, or _____. My favorite method is to _____. Sometimes I have to _____ because _____.

The next day in class, have students in groups read other group members' essays. Groups can choose one essay to read or present to the class. Students can also select a procedure that a class member wrote about and apply it to a problem.

Standard 3: Mathematics as Reasoning

In grades 9–12, the mathematics curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can—

- make and test conjectures;
- formulate counterexamples;
- follow logical arguments;
- judge the validity of arguments; and
- construct simple valid arguments.

Critical thinking is the heart of mathematics. In fact, mathematical problems cannot be solved without reasoning the solution process and deciding if the answer is logical. This standard does not refer to the formal reasoning process but it should portray the conjecturing, validating, refuting, and analyzing processes that are used to make sense of mathematics. Students can use various means to communicate their reasoning, such as oral discussion, the construction of models, and written explanations.

Standard 3: Reasoning Activity

Standard's bullets addressed by activity

- *make and test conjectures*
- *follow logical arguments*
- *judge the validity of arguments*

Materials needed

- Icosahedral dice (20 sides, 0–9 appearing twice)

Technology

- Calculators

Description of Activity

- **Group or classroom management practices**

Students should work in groups.

Suppose Rowena tells you that under her old method of shooting free throws in basketball, her average was 60%. Using a new method of shooting, she scored 9 out of her first 10 throws. Should she conclude that the new method really is better than the old method? All students should first identify the real (statistical) question: What are the chances of shooting at least nine out of ten if you normally shoot 60%?

Each group could model the problem by associating baskets with die sides 4–9 inclusive and misses with sides 0–3 inclusive. A trial is defined as 10 rolls of a die. If 9 or more "baskets" occur, count the trial as a success. Class discussions about the number of trials needed and the level of confidence associated with their conclusions should be included.

Reference

National Council of Teachers of Mathematics, (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

Standard 4: Mathematical Connections

In grades 9–12, the mathematics curriculum should include investigation of the connections and interplay among various mathematical topics and their applications so that all students can—

- recognize equivalent representations of the same concept;
- relate procedures in one representation to procedures in an equivalent representation;
- use and value the connections among mathematical topics; and
- use and value the connections between mathematics and other disciplines.

Mathematics is often thought of as separate topics that are not intertwined but are studied in isolation. Connecting the topics helps students form a more cohesive knowledge base. This eliminates the memorization of facts or procedures that appear to have little meaning.

Connections also relate directly to how students learn mathematics. Students who make connections usually explore a mathematical topic for a period of days and then bridge ideas that are represented in concrete form to a more abstract one. Additionally, they are able to discern similarities and differences among ideas and procedures.

Standard 4: Mathematical Connections Activity

Standard's bullets addressed by activity

- *recognize equivalent representations of the same concept*
- *relate procedures in one representation to procedures in an equivalent representation*
- *use and value the connections among mathematical topics*

Technology

- Calculators could be used as a tool for a graphing method to a solution.

Description of Activity

- **Group or classroom management practices**

Small groups of three to four students are recommended.
Present the following problem: Sally had 27 coins totaling $2.30. If she has only dimes and nickels, how many does she have of each type?

Have students solve the problem in as many different ways as their group can. (The problem can be solved by a system of equations method, guess-and-test, a table or chart, and matrices.) Discuss different solving strategies and the advantages/disadvantages of each method.

- **Extensions**

Have groups write their own problems, similar to this one, that can be solved with multiple-solution methods. Have groups present problems to the class for discussion.

Standard 5: Algebra

In grades 9–12, the mathematics curriculum should include the continued study of algebraic concepts and methods so that all students can—

- represent situations that involve variable quantities with expressions, equations, inequalities, and matrices;
- use tables and graphs as tools to interpret expressions, equations, and inequalities;
- operate on expressions and matrices, and solve equations and inequalities; and
- appreciate the power of mathematical abstraction and symbolism.

Algebra has typically been thought of as very abstract, with the outcome being that students solve equations in a very prescribed step-by-step way. The notion of what algebra represents is changing. Algebra is finding patterns and then describing those in a variety of ways.

In high school grades, students should begin to move to a more formal level but again, there should be an emphasis on procedures that are derived from students' thinking rather than those prescribed by a textbook. There is less of a focus on strict procedural aspects of algebra than in a traditional curriculum. Instead, students continue to use patterns as a basis for algebraic manipulations and methods.

Tools for algebra should include a graphing calculator and computers, as they become available. Many routines that were long regarded as necessary for algebra can now be done using technology. It is thus important that students understand how to assess reasonableness of responses.

Standard 5: Algebra Activity

Standard's bullets addressed by activity

- *use tables and graphs as tools to interpret expressions, equations, and inequalities*
- *operate on expressions and matrices, and solve equations and inequalities*

Materials needed

- Graph paper (optional)

Technology

- Calculators optional

Description of Activity

- **Group or classroom management practices**

  Students can work individually or in pairs to solve the problem.

A football player attempts to make a field goal by kicking from the 20 yard line. The ball is placed 30 yards from the goal posts. The goal post's cross bar is 5 yards above the ground. When kicked, the ball reaches its highest altitude of 32 feet at a point 48 feet from where it was kicked.

a) Make a sketch showing the path of the ball. Describe the path.

b) If the point from which the ball was kicked is the origin of a coordinate system, find an equation for the path of the football.

c) Will the kicker make the field goal?

d) Suppose the offensive team was given a penalty and sent back 5 yards. Given that the highest altitude again is 32 feet at a point 48 feet from where it was kicked, write a new equation. Will the kicker make the goal?

Standard 6: Functions

In grades 9–12, the mathematics curriculum should include the continued study of functions so that all students can—

- model real-world phenomena with a variety of functions;
- represent and analyze relationships using tables, verbal generalizations, equations, and graphs;
- translate among tabular, symbo[illegible], and graphical representations of functions; and
- classify, compare, or contrast a variety of problem situations that can be modeled by the same type of function.

Students began an informal study of functions in middle grades and should continue those investigations into high school grades. Functions are a natural way to describe relationships in multiple contexts. For example, there can be numerical functions, variable relationships found in algebra, and transformations in geometry. Each of these contexts can be found in real-world situations.

Standard 6: Functions Activity

Standard's bullets addressed by activity

- *analyze the effects of parameter changes on the graphs of functions*
- *understand operations on, and the general properties and behavior of, classes of functions*

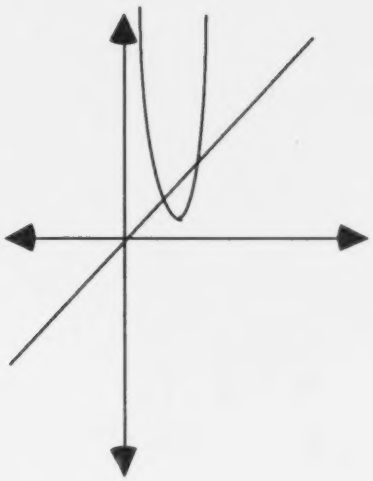
Technology

- Computer with appropriate graphing software or graphing calculator is **necessary** for this activity.

Description of Activity

- **Group or classroom management practices**

  Students can work in groups of one to four students.



Consider this graph of a curve and the line $y = x$. Assume the curve has the equation:

$$y = (x - r)^2 + r$$

Using a graphing calculator, discover possible values for the boxes which would provide a graph similar to the one given.

What values provide a similar graph?
What values do not provide a similar graph?

- **Extensions**

What is the significance of h and k in the equation $y = (x - h)^2 + k$? What is the significance of c in the equation $y = c(x - h)^2 + k$?

Standard 7: Geometry

In grades 9–12, the mathematics curriculum should include the continued study of geometry of two and three dimensions so that all students can—

- interpret and draw three-dimensional objects;
- represent problem situations with geometric models and apply properties of figures;
- classify figures in terms of congruence and similarity and apply these relationships; and
- deduce properties of, and relationships between, figures from given assumptions.

Geometry is a basic skill for all citizens. Real-world situations, in fact, are the bases for building geometric concepts and ideas. Geometry also spans other disciplines, such as science and the arts; and connects mathematical topics, such as fractions with area models.

While proof has typically dominated the geometry curriculum, there are many types of proof, including paragraph, oral, and diagrammatic. Students should not be held to the two-column proof done "by the book" but should be encouraged to apply logical thinking to geometric contexts.

Standard 7: Geometry Activity

Standard's bullets addressed by activity

- *interpret and draw three-dimensional objects*
- *represent problem situations with geometric models and apply properties of figures*

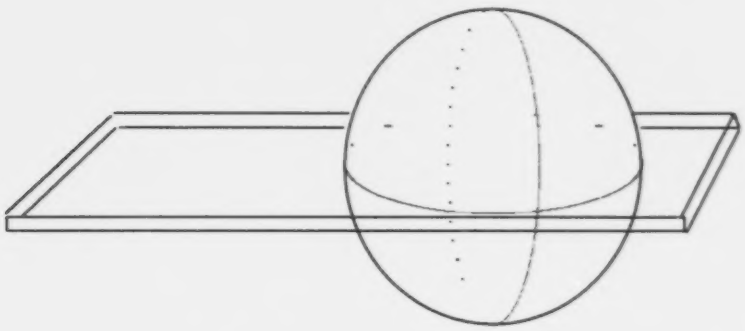
Description of Activity

- **Group or classroom management practices**

  Students work in small groups of three to four students.

The Waikiki Aquarium is designing some new fish tanks which are to be spherical in shape. They will fit into holes in a Plexiglas base in such a manner that the entire

sphere will be visible. There will be three spherical tanks that will fit into the base. What should be the diameter of the holes if the diameter of the sphere is 36" and must fit so that $\frac{1}{3}$ of the sphere is below the Plexiglas base?



- **Extensions**

The durability of the Plexiglas was tested and because of the weight of the aquariums, the largest diameter of a hole that could be cut is 11 inches. How much of the aquarium is below the glass?

Reference

American High School Mathematics Examination, 1987.

Standard 8: Geometry from an Algebraic Perspective

In grades 9–12, the mathematics curriculum should include the study of the geometry of two and three dimensions from an algebraic point of view so that all students can—

- investigate multiple representations of geometric phenomena;
- deduce properties of figures using transformations and using coordinates;
- identify congruent and similar figures using transformations;
- analyze properties of Euclidean transformations; and
- relate translations to vectors.

There have long been connections made between geometry and algebra. While coordinates have been the typical manner by which these connections were made, transformations have entered the contemporary view of geometry. By adding motions, students can now investigate geometry ideas while forming a basis for vectors.

Standard 8: Geometry from an Algebraic Perspective Activity

Standard's bullets addressed by activity

- *deduce properties of figures using transformations and using coordinates*
- *identify congruent and similar figures using transformations*
- *analyze properties of Euclidean transformations*

Materials needed

- Graph paper

Description of Activity

- **Group or classroom management practices**

  Students can work individually or in small groups of two to four students.

Given A(3, 3), B(5, 1), and C(8, 1), find D such that ABCD is an isosceles trapezoid. Find another D such that ABCD is a different isosceles trapezoid.

Discuss similarities and differences in the trapezoids formed by the selection of different coordinates. How many possible solutions are there?

- **Extension**

Suppose the four vertices of an isosceles trapezoid, when graphed on the Cartesian coordinate plane, are located in Quadrant IV. Also, you know the slope of the parallel sides is $\frac{1}{2}$ and point A(1, -2) and C(5, -3) are located on opposite parallel sides. Find B and D. Explain how you know your B and D are located at correct coordinates.

Standard 9: Trigonometry

In grades 9–12, the mathematics curriculum should include the study of trigonometry so that all students can—

- apply trigonometry to problem situations involving triangles;
- explore periodic real-world situations using trigonometric functions;
- use circular functions to model periodic real-world phenomena;
- apply graphing techniques to trigonometric functions; and
- solve trigonometric equations and verify trigonometric identities with appropriate technology.

Trigonometric contexts can involve many real-world settings and should stem from the study of triangle measurements. Navigation, part of our island cultures, is a natural setting for exploring rotations, direction angles of vectors, and polar coordinates.

Both trigonometric and circular functions should be investigated from the generalizations of triangle trigonometry. Graphing tools would enhance the studies.

Standard 9: Trigonometry Activity

Standard's bullets addressed by activity

- *apply graphing techniques to trigonometric functions*
- *solve trigonometric equations and verify trigonometric identities with appropriate technology*

Technology

- Graphing calculators

Description of Activity

- **Group or classroom management practices**

  Groups of three to four students.

In this activity, students will graph equations to determine if the equation is or is not an identity. Have students graph each side of the equation on the same graph. Is the equation an identity? Ask students to explain how the graph helps determine if the equation is or is not an identity. For example, $\sin^2 x + \cos^2 x = 1$. Graph $y = \sin^2 x + \cos^2 x$ and $y = 1$ on the same graph. Is $\sin^2 x + \cos^2 x = 1$ an identity? How do you know?

$\cos 2x = \cos^2 x - \sin^2 x$
$\cos 2x = 2(\sin 2x) - 1$
$1 + \tan^2 x = \sec^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$
$\tan x \cos x = 1$

- **Extension**

Describe other ways besides graphing that you can use to prove that an equation is/is not an identity. Provide examples or counterexamples.

Standard 10: Statistics

In grades 9–12, the mathematics curriculum should include the continued study of data analysis and statistics so that all students can—

- construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations;
- use curve fitting to predict from data;
- understand and apply measures of central tendency, variability, and correlation;
- understand sampling and recognize its role in statistical claims;
- design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes; and
- analyze the effects of data transformations on measures of central tendency and variability.

Statistics is the means by which we can describe data. A study of statistics is not just of formulas but explorations about how to display the data or what means to use to describe it. Students should explore statistical ideas and develop statistical techniques from their observations and generalizations.

Standard 10: Statistics Activity

Standard's bullets addressed by activity

- *construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations*
- *understand sampling and recognize its role in statistical claims*
- *design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes*

Materials needed

- Graph paper

Technology

- **Calculators**

Description of Activity

- **Group or classroom management practices**

  Students should work in groups of at least two students.

Have students investigate possible relationships between the age and mileage of cars by collecting data from the school parking lot or nearby towns. Students construct a scatter plot, generating an equation for the regression line and then use either their graph or their equation to predict (with a measurable certainty), for example, the expected mileage of a 1980 car. What other relationships might be possible to find, using a car's age as one factor?

Reference

National Council of Teachers of Mathematics, (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

Standard 11: Probability

In grades 9–12, the mathematics curriculum should include the continued study of probability so that all students can—

- use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty;
- use simulations to estimate probabilities;
- create and interpret discrete probability distributions; and
- describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed.

Probability is the study of predictions because information about probability helps us make decisions. Probability explorations are closely woven with statistical methods. Students should be encouraged to use appropriate technology and other tools to make their predictions. Probability explorations should include investigations of a normal curve as well as the selection process for an appropriate distribution.

Standard 11: Probability Activity

Standard's bullets addressed by activity

- *use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty*
- *use simulations to estimate probabilities*

Description of Activity

- **Group or classroom management practices**

  Groups of three to four students

Suppose there were six papers selected for the final round of a competition. Five judges each rated their top three papers as 1, 2, and 3, with 1 as the best paper. The results were as follows:

Paper A 2, 1, 3, 2, 2
Paper B 3
Paper C 1, 1, 1, 3

Paper D 2, 3, 2
Paper E 1, 3
Paper F [did not place]

If first place receives $5,000 and second place receives $1,000, which two papers do you think should receive the prizes and why? Be prepared to defend your decision.

- **Extensions**

Pose the following questions:

- How could you guarantee more consistent ratings?
- How would it affect your decision if *each* paper were ranked 1–6?

Reference

Burrill, G., Burrill, J. C., Coffield, P., Davis, G., Lange, J. D., Resnick, P., & Siegel, M. (1992). *Data analysis and statistics across the curriculum.* Reston, VA: NCTM. p. 32.

Standard 12: Discrete Mathematics

In grades 9–12, the mathematics curriculum should include topics from discrete mathematics so that all students can—

- represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations;
- represent and analyze finite graphs using matrices;
- develop and analyze algorithms; and
- solve enumeration and finite probability problems.

Discrete mathematics allows students to explore unique problem situations that are not directly approachable through writing an equation or applying a common formula. Problems illustrating discrete mathematics can be classified in three broad categories. The first category, *existence problems*, deals with whether a given problem has a solution or not. The second category, *counting problems*, investigates how many solutions may exist for problems with known solutions. A third category, *optimization problems*, focuses on finding the best solution to a particular problem. (NCTM, 1991, p. 2)

Discrete mathematics provides the opportunity to make mathematical connections among other topics because its related problems are real-world oriented. There is a need for technology to be applied in the problem settings. Students will continue to develop their critical thinking and mathematical reasoning skills as they work through discrete mathematics problems.

Standard 12: Discrete Mathematics Activity

Standard's bullets addressed by activity

- *develop and analyze algorithms*
- *solve enumeration and finite probability problems*

Technology

- The use of calculators (one for each student) is highly recommended.

Description of Activity

- Group or classroom management practice

  Students should work in small groups of three to four members.

A fisherman wished to return home at the end of the day with his cooler full of valuable fish. This fisherman had a particularly good day in that the amount of fish caught exceeded the capacity of his one and only ice chest. He must decide which fish should be packed in his ice chest and which fish should be given away. His goal is to sell the fish in his ice chest for as much money as possible.

How should he pack his ice chest? Justify your answer in writing and explain the method or algorithm used.

Ice chest dimensions: 2 feet wide, 3 feet long, 2 feet high, 1 cubic foot of ice in the cooler

YELLOW-FIN TUNA	MAHI-MAHI	WAHOO	GROUPER
5 fish caught	7 fish caught	2 fish caught	24 fish caught
Sells for $6 per pound	Sells for $4 per pound	Sells for $9 per pound	Sells for $3 per pound
Each fish weighs 12 pounds.	Each fish weighs 9 pounds.	Each fish weighs 7 pounds.	Each fish weighs 3 pounds.
Volume of each fish is 4 Ft^3	Volume of each fish is 2 Ft^3	Volume of each fish is 1.5 Ft^3	Volume of each fish is 1 Ft^3

- **Extension**

The weight and volume of each individual fish is specified.

	Weight	Volume
Fish 1	10 lb.	4 Ft^3
Fish 2	5 lb.	6 Ft^3
Fish 3	20 lb.	5 Ft^3
Fish 4	7 lb.	5 Ft^3
Fish 5	12 lb.	10 Ft^3

Reference

Kenney, M. J., and Hirsch, C. R. (Eds.) (1991). *Discrete mathematics across the curriculum, K–12*. Reston, VA: National Council of Teachers of Mathematics.

Standard 13: Conceptual Underpinnings of Calculus

In grades 9–12, the mathematics curriculum should include the informal exploration of calculus concepts from both a graphical and a numerical perspective so that all students can—

- investigate limiting processes by examining infinite sequences and series and areas under curves;
- analyze the graphs of polynomial, rational, radical, and transcendental functions;
- determine maximum and minimum points of a graph and interpret the results in problem situations; and
- understand the conceptual foundations of limit, the area under a curve, the rate of change, and the slope of a tangent line, and their applications in other disciplines.

This standard does *not* advocate the formal study of calculus in high school for all students or even for college-intending students. Rather, it calls for opportunities for students to systematically, but informally, investigate the central ideas of calculus—limit, the area under a curve, the rate of change, and the slope of a tangent line—that contribute to a deepening of their understanding of function and its utility in representing and answering questions about real-world phenomena.

Standard 13: Conceptual Underpinnings of Calculus Activity

Standard's bullets addressed by activity

- *determine maximum and minimum points of a graph and interpret the results in problem situations*
- *understand the conceptual foundations of limit, the area under a curve, the rate of change, and the slope of a tangent line, and their applications in other disciplines*

Technology

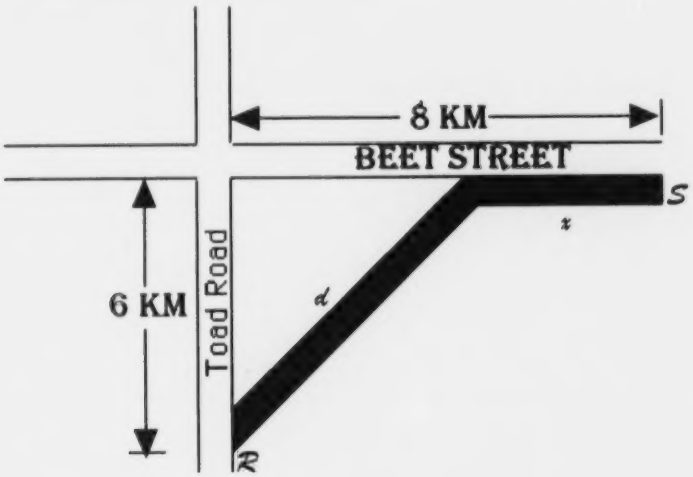
- Graphing calculators

Description of Activity

- **Group or classroom management practices**

  Students should work in groups of three to four students.

Motivating question: A pipeline is to be built from a point R to a point S as shown on the map. Points R and S are connected by roads as shown.



The pipeline will be built either across the land to a point x kilometers from S and then along Beet Street to S, as shown in the sketch, or entirely along Toad Road and Beet Street. If the cost is \$12,000 a kilometer along the road or street and \$37,000 a kilometer across the land, find x so that the cost will be minimal.

1. How much will it cost to build:
   a. 10 km of pipeline along the road? [\$120,000]
   b. 10 km of pipeline across the land? [\$370,000]
   c. 3.5 km of pipeline along the road and 7.5 km of pipeline across the land? [\$319,500]

2. How much will it cost to build the pipeline when x is:
   a. 0 km? [\$370,000]
   b. 2 km? [\$337,955]
   c. 4 km? [\$314,811]
   d. 6 km? [\$306,009]

3. Write a formula for the cost when the pipeline is built entirely along Toad Road and Beet Street. [C = \$168,000]

Reference

Meiring, S. P., Rubenstein, R. N., Schultz, J. E., Lange, J. D., & Chambers, D. L. (1992). *Curriculum and evaluation standards for school mathematics addenda series. Grades 9-12: A core curriculum.* Reston, VA: NCTM. pp. 95-96.

Standard 14: Mathematical Structure

In grades 9–12, the mathematics curriculum should include the study of mathematical structure so that all students can—

- compare and contrast the real number system and its various subsystems with regard to their structural characteristics; and
- appreciate that seemingly different mathematical systems may be essentially the same.

Mathematical structure in the form of lists of general properties is not a good starting point for instruction. Rather, students gain a sense of the structure of mathematics over an extended time period through the general accumulation of experience, as well as through more focused activities. It is neither necessary nor appropriate for them to hear constantly the word **structure** applied to their activities; occasional summary statements will serve them far better. It also is essential to recognize that mathematical structure and formalism are not synonymous. In mathematics, just as with a building, students can develop an understanding and appreciation of its underlying structure independent of a knowledge of the corresponding technical vocabulary and symbolism. The degree of formalism must be consistent with the students' level of mathematical maturity.

Standard 14: Mathematical Structures Activity

Standard's bullets addressed by activity

- *compare and contrast the real number system and its various subsystems with regard to their structural characteristics*
- *appreciate that seemingly different mathematical systems may be essentially the same*

Materials needed

- Graph paper

Description of Activity

- **Group or classroom management practices**

  Groups of two to four students

Students will write a paragraph comparing a mathematical structure that they have studied with a structure in their life experience.

Example: The first homes people lived in were caves. Soon they felt a need to have homes in different areas, and so, they built structures with roofs and sides. More people in the family soon required more rooms in the house. Clothing made closets necessary. Bathrooms were added. Windows made houses more comfortable and pleasant. Houses became more elaborate to meet the needs of the people living in them.

Similarly, mathematics began with counting numbers. Soon negative values were needed, and mathematics included the integers. Parts of wholes were needed, and so, fractions developed. Irrational numbers filled the holes on the number line. To solve problems like $x^2 = -9$, imaginary numbers were created. All of these numbers fall under the umbrella called the "complex-number system."

Students should write and illustrate or model the commutative property of addition using natural numbers integers, irrational numbers, complex numbers, and matrices.

- **Extensions**

Students can present their paragraphs to the class. Possible questions for the class include:

- Are there connections which could be included that were not mentioned?
- Does the illustration hold true for the commutative property of multiplication? In which sets of numbers?

EPILOGUE

NEXT STEPS: OUR CHALLENGE

Our challenge now is to head towards change in as many steps and paths as needed but all leading in the same direction. All educators, from teachers to administrators to college professors, will follow different roads to systemic change in school mathematics. Some of these paths follow.

Curriculum Development

The *Pacific Standards for Excellence in Mathematics* will drive curriculum revisions from traditional scope and sequence courses that treat mathematics as a body of isolated concepts and procedures to integrated mathematics topics across content areas. Curriculum development moves toward connecting mathematics, its ideas and its applications throughout all discipline areas.

This document provides a basis for an evolving curriculum, one that melds new information into existing practice. The result is continual movement toward enhanced student understanding through effective teaching and assessment strategies.

Textbooks and Other Materials

In the traditional school system, mathematics textbooks serve as the core of mathematics programs. Textbooks currently used in our schools do not meet the *Pacific Standards*, even though we attempt to revise the lessons to do so. Instead, other materials that support the *Standards*, such as manipulatives and software, must be developed in addition to new textbooks.

Instruction

Mathematics literacy is for all students, not just for those with special abilities or for those going on to college. To meet literacy needs of the future, students must be able to solve mathematical problems and to communicate mathematically. Instruction takes place in the classroom, and the teacher is the key agent. As the focus of instruction shifts from the teacher as the sole source of knowledge to teacher as facilitator of student-centered, problem-solving-based learning, teachers must buy into the changing focus. Instructional reform will not occur unless it comes from the teachers. The class, including the teacher, becomes a learning community where ideas are shared as teacher and students learn together.

The teacher's role changes dramatically from dispenser of information to guide and facilitator of learning by promoting open-ended inquiry, introducing concepts through

problems, and allowing students time to develop concepts, generalizations, and skills. The teacher-as-learner is an important part of the process. Teachers become listeners who incorporate student thinking into their instruction.

Students take on an active role in their learning process. They no longer passively memorize rules and formulas in order to find answers, but they develop skills in problem-solving processes. Students learn best as they explain their thinking when they construct their own understanding of concepts and generalizations. Class time is spent on understanding the whys and hows of mathematics. In this context, learning includes the opportunity to explore and investigate many answers and a variety of solutions.

The traditional class period of 10 minutes for grading homework, 20 minutes of teacher lecture to introduce the day's new topic, and 15 minutes for students to start that evening's homework is replaced by a new class format. Students work collaboratively to solve problems and develop justifications and validations for their thinking. They then present their solutions to the class.

In a collaborative environment, students bounce their ideas off others. They must, therefore, each take the responsibility to justify their own thinking, as well as to constructively comment on others' ideas. As students become accustomed to a collaborative environment, they learn more mathematics, take greater pride in their own work, respect others' thinking, and feel more comfortable talking about mathematics.

Alternate Assessments

With the shift of classroom instruction to a focus on student thinking, cooperative/collaborative groups, open-ended investigations, and learning through interactions, classroom assessment must also change. It should reflect the recommendations of the *Pacific Standards* in both mathematical content and instructional approach. That is, assessment tasks should encompass multiple ways for students to express their understandings.

Teacher In-Service Programs

Teachers in our region may be ready to teach the kind of mathematics program outlined in the *Pacific Standards*. Many others, however, may need additional inservice and refresher courses. Such staff development activities must be formulated in collaboration with the teachers who understand the required systemic changes and who can form links between staff development and classroom practices. To ensure support for the proposed changes, administrators must participate in all staff development efforts.

Teacher Preservice Education

Colleges and universities should also work with schools to redesign teacher preparation programs to ensure that teachers are prepared to teach in ways aligned with the *Pacific Standards*. All teachers need an understanding of mathematical concepts and relationships among them and other disciplines. Teachers must also experience the learning of mathematics through instructional approaches consistent with those to be used in their own classrooms.

Technology

Living in a technological world, our students need to be familiar with the power of technology. Mathematics courses must incorporate appropriate technology such as calculators, computers, interactive television, and multimedia into the instructional approach.

Concluding Remarks

The *Pacific Standards for Excellence in Mathematics* can serve as the basis for improving the teaching and learning of mathematics in our Pacific schools. Parental, community, and professional interest and concern, when combined with changing technology and a growing body of research-based knowledge, are the ingredients necessary for genuine systemic reform based on the *Standards*.

At the forefront of such reform are the members of the Mathematics Leadership Team, representing each Pacific entity. Under the team's leadership, teachers of American Samoa, Commonwealth of the Northern Mariana Islands, Federated States of Micronesia (Chuuk, Kosrae, Pohnpei, and Yap), Guam, Hawaii, Republic of the Marshall Islands, and the Republic of Palau in every mathematics classroom will know and use the *Standards* in guiding their instruction. Thus, the vision becomes reality: "All Pacific children will be scientifically and mathematically literate: knowledgeable, capable, and caring."

OBTAINING MORE INFORMATION ABOUT THE STANDARDS

Further information regarding the *Pacific Standards for Excellence in Mathematics* and other documents in the Pacific Standards for Excellence Series can be obtained from:

The Pacific Mathematics and Science Regional Consortium
PACIFIC REGION EDUCATIONAL LABORATORY
828 Fort Street Mall - Suite 500
Honolulu, Hawaii 96813
tel: 808-533-6000
e-mail: askmathsci@prel.hawaii.edu

or the Mathematics Leadership Team Members listed below:

AMERICAN SAMOA
Mr. Philo Jennings
Division of Curriculum & Instruction
Department of Education
P.O. Box 3234
Pago Pago, AS 96799
Phone: (684) 633-1246
Fax: (684) 633-5184

CNMI
Mr. Nicolas D. Sablan
Public School System
P.O. Box 1370
Saipan, MP 96950
Phone: (670) 288-0690
Fax: (670) 322-4056

CHUUK
Mr. Sanfio Sony
Department of Education
P.O. Box 460
Weno, Chuuk FM 96942
Phone: (691) 330-2202/2203/2475
Fax: (691) 330-2604

GUAM
Ms. Teresita Mesa
Public School System
P.O. Box 6264
Tamuning, GU 96931
Phone: (671) 475-0454
Fax: (671) 472-9750

HAWAI'I
Ms. Jeanne Nelson
Kamehameha Schools
Kapalama Heights
Honolulu, HI 96817
Phone: (808) 842-8924
Fax: (808) 842-3927

Ms. Kathleen Nishimura
Department of Education
OIS, GEB, Math & Science Section
189 Lunalilo Home Road, 2nd Flr.
Honolulu, HI 96825
Phone: (808) 396-2567
Fax: (808) 548-5390

KOSRAE
Mr. Gibson Mongkeya
Department of Education
P.O. Box ED
Tofol, Kosrae FM 96944
Phone: (691) 370-3189/3208
Fax: (691) 370-2045

PALAU
Mr. Tangy C. Anastacio
Mr. April Olkeriil
Ministry of Education
P.O. Box 189
Korror, PW 96940
Phone: (680) 488-2952
Fax: (680) 488-2830/1465

RMI
Mr. Bedinin Joseph
Ministry of Education
Box 3
Majuro, MH 96960
Phone: (692) 625-5261/2
Fax: (692) 625-3861

POHNPEI
Mr. Pemis Diopulos
Ms. Schliter Joseph
Department of Education
P.O. Box 304
Kolonia, Pohnpei FM 96941
Phone: (691)320-2146
Fax: (691) 320-5510

YAP
Ms. Ginny Fenenigog
P.O. Box 220
Colonia, Yap FM 96943
Phone: (691) 350-2146
Fax: (691) 350-2399

REFERENCES

Dougherty, B. J. (1993, Summer). Collaborative or cooperative groups: That is the question. *Voyages in Mathematics and Science*, Issue 4.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (1991). *Mathematics assessment: Myths, models, good questions, and practical suggestions*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics*. Reston, VA: NCTM.

National Science Foundation (February, 1993). *Excellence in mathematics and science education, K-16*. Proceedings from the National Science Invitational Conference, Washington, D. C.

Rachlin, S. L., Matsumoto, A., and Wada, L. (1992). *Algebra I: A process approach*. Honolulu, HI: University of Hawaii, Curriculum Research and Development Group.

PACIFIC REGION EDUCATIONAL LABORATORY
828 Fort Street Mall ◆ Suite 500
Honolulu, Hawaii 96813-4321
(808) 533-6000 ◆ FAX: (808) 533-7599
e-mail: askprel@prel.hawaii.edu

BEST COPY AVAILABLE